Generalizations of Opial’s theorem and the common fixed point problems

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Let a sequence \( \{x^k\}_{k=0}^{\infty} \subset \mathcal{H} \) be generated by a recurrence \( x^{k+1} = T_k x^k \), where \( x^0 \in \mathcal{H} \) and \( \{T_k\}_{k=0}^{\infty} \) is a sequence of operators defined on a Hilbert space \( \mathcal{H} \). The Opial theorem says that if \( T_k = T \) for all \( k \geq 0 \) and \( T \) is nonexpansive and asymptotically regular, then \( x^k \) converge weakly to a point \( x^* \in \text{Fix} T \). We present a generalization of this Theorem to the case where each iteration can employ different operators \( T_k \) having a common fixed point, which are not supposed to be nonexpansive. We also present several applications of this Theorem to the common fixed point problem.