Generalized relaxations of nonexpansive operators with applications to convex optimization problems

Andrzej Cegielski
Faculty of Mathematics, Computer Science and Econometrics
University of Zielona Góra, Poland
e-mail: a.cegielski@wmie.uz.zgora.pl

Abstract

Let \( X \subseteq H \) be a nonempty, closed and convex subset of a real Hilbert space \( H \) and let \( T : X \to X \) be a nonexpansive operator with nonempty subset of fixed points. We introduce a generalized relaxation \( T_{\sigma,\lambda} : X \to X \) of \( T \) in the form
\[
T_{\sigma,\lambda}(x) = x + \lambda \sigma(x)(Tx - x),
\]
where the relaxation parameter \( \lambda \in [0, 2] \) and the step size function \( \sigma : X \to (0, +\infty) \). We propose some conditions on which the sequence \((x_k)\) defined by the recurrence \( x_{k+1} = T_{\lambda_k,\sigma}(x_k) \) converges weakly to a fixed point of \( T \) for arbitrary starting point \( x_0 \in X \). We also present applications of the results to convex optimization problems.