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Sensors' allocation for estimating scalar fields by wireless sensor networks

Summary

- Introduction
- Problem statement
- Optimality conditions
- Searching for optimal allocations – description of the algorithm
- Examples

Introduction

- **We consider the problem of allocating sensors and frequencies of their activation**
- **in such a way that the balance between the estimation accuracy of a scalar field and energy consumption is optimized.**
- **We propose an extension of the theory of optimal experiment designs by including terms for transmission energy consumption or penalties for region coverage into the optimality criterion.**

Introduction 2

- We state the optimality conditions, which serve as a base for a numerical algorithm for searching sensors' positions and frequencies of their activations.
- Examples of optimal allocations for a variety of basis, spanning spatial fields, are also provided.

Introduction 3

- Applications of wireless sensors networks (WSN) are so wide that we have to consider them in a problem dependent way,
- i.e., sensors' locations for different tasks should be different – there is no one universal scheme.
- In this paper we confine our discussion to a subclass of WSN's, which are specialized for estimating scalar spatial fields.

Introduction 4

Possible applications of WSN's for estimating scalar fields include:

- monitoring of environmental conditions: temperature, humidity, pressure, rainfall, chemical contamination and many others.
- diagnosis of machine parts, chemical reactors, bridges, pipelines, vibrations etc.
- In the above applications it is possible and economically plausible to allocate sensors carefully, taking into account their tasks as well as power consumption for data transmission.

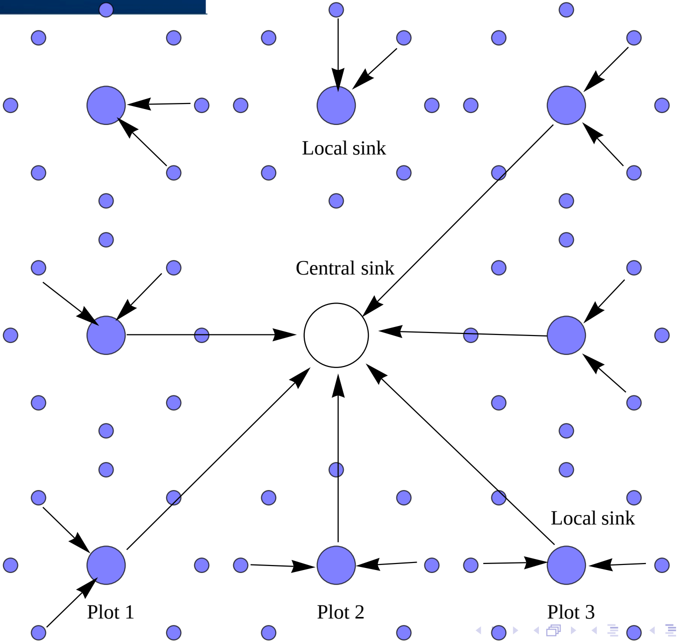
Introduction 5

- A dense network of randomly deployed and cheap sensors also allows spatial sampling, but – if applicable – a careful sensors' location provides more information and lower power consumption.
- The main goals: to cover all the area, to ensure connectivity (so as to assure data transmission between nodes).
- It still remains flexibility for goal-oriented sensors' allocation. Such point of view is not common in the papers on WSN's design (see the bibliography).

Introduction 6

- The idea of using an optimal experiment design setting for sensors' allocation can be traced back to early 80's.
- The main difference: here we explicitly take into account energy consumption for transmission in WSN's, incorporating it into the goal function.
- The area is divided into plots (squares, circles hexagons), which form a tilling.

Network structure 1



Network structure 2

- sensors (smaller blue circles),
- local sinks (larger blue circles), which are simultaneously sensors and local processing units, sufficiently powerful to solve a standard LSQ estimation problem (described below).
- The largest circle is a central sink.
- Arrows = a wireless data transmission.
- Data transmitted from a sensor to a local sink = measurements, (initially processed – verification of ranges, averaging several measurements etc.).

Problem statement 1

- Sensors' allocation in one, typical, plot is optimized and replicated.
- Sensors located in each plot perform measurements with prescribed frequencies.
- Both sensors' locations x_i's and frequencies $p_i > 0$, $\sum_{i=1}^{m} p_i = 1$ are our decision variables, collected in a design ξ: it is convenient represent ξ as the table:

$$\xi = \begin{bmatrix} x_1 & x_2 & \ldots & x_m \\ p_1 & p_2 & \ldots & p_m \end{bmatrix} \qquad (1)$$

Problem statement 2

Observations (simplified – see the paper):

$$y_i = a^T v(x_i) + \epsilon_i, \quad i = 1, 2, \ldots, n, \qquad (2)$$

ϵ_i's uncorrelated random errors, $E(\epsilon_i) = 0$, $a \in R^d$ – unknown – our goal=to estimate them, $v(x)$ – selected basis functions, e.g., – bivariate polynomials of the second order:

$$v^T(x) = [1, x^{(1)}, x^{(2)}, x^{(1)} x^{(2)}, \qquad (3)$$

$$x^{(1)} (x^{(2)})^2, (x^{(1)})^2 x^{(2)}, (x^{(1)})^2, (x^{(2)})^2, (x^{(1)})^2 (x^{(2)})^2]$$

– radial basis functions,
– trigonometric polynomials, splines etc.

Problem statement 3

- Parameters a are estimated by the least LSQ, which provides linear (in y_i's), unbiased estimator $\hat{a}$.
- Its covariance matrix has the form: $cov(\hat{a}) \sim M^{-1}(\xi)$, where
- $M(\xi)$ – the normalized Fisher's information matrix:

$$M(\xi) = \sum_{j=1}^{m} p_j \, v(x_j) \, v^T(x_j). \tag{4}$$

- Thus, the estimation accuracy depends on senors' allocation x_j's and on relative frequencies of their usage p_j's.

Problem statement 4

- It is customary to minimize D-optimality $\ln(\mathrm{Det}(M^{-1}(\xi)))$, since Det of $M^{-1}(\xi) \sim$ vol. of the uncertainty ellipsoid of $\hat{a}$. Equiv. to maximize $\ln(\mathrm{Det}(M(\xi)))$.
- Let $d(x, x_0)$ be a user defined function, which takes into account specific features of data transmission in WSN's. E.g.,

$$d_1(x, x_0) = \gamma \left((x_0^{(1)} - x^{(1)})^2 + (x_0^{(2)} - x^{(2)})^2 \right)^{\beta}$$

is the energy consumption for transmission from sensor at x to sink at x_0, where $\gamma > 0$ and $\beta > 0$ are case dependent constants.

Problem statement 5

For the coverage problem d is of the form:

$$d_2(x,\ x_0) = \lambda\,[((x_0^{(1)} - x^{(1)})^2 + (x_0^{(2)} - x^{(2)})^2)^{1/2} - \rho]^2$$

a penalty for a sensor at x to be too far or too close to sink x_0, ρ a desired radius.
A penalty for being too far from x_0:
$d_3(x,\ x_0) = \lambda\ \exp\left[-\frac{||x - x_0||^2}{2\,\sigma^2}\right]$, where $||.||$ – the Euclidean norm, $\sigma > 0$ a spread parameter.

- Clearly, one can mix:
  $d(x,\ x_0) = d_1(x,\ x_0) + d_2(x,\ x_0)$ or
  $d(x,\ x_0) = d_1(x,\ x_0) + d_3(x,\ x_0)$.

Problem statement 6

As the goal function we take:

$$F(\xi) = \ln(\mathrm{Det}(M(\xi))) - D(\xi), \text{ where} \tag{5}$$

$$D(\xi) \stackrel{\text{def}}{=} \sum_{j=1}^{m} p_j \, d(x_j, x_0). \tag{6}$$

- The first term in (5) takes into account the estimation error.
- $D(\xi)$ is the weighted sum of energy for transmitting observations from sensors placed at x_i's to the sink at x_0.

Problem statement 7

- Our aim is to find ξ^*, which <u>maximize</u> $F(\xi)$ over all ξ's such that $p_j \geq 0$, $\sum p_j = 1$:

$$\xi = \begin{bmatrix} x_1 & x_2 & \dots & x_m \\ p_1 & p_2 & \dots & p_m \end{bmatrix}, \quad x_j \in X\text{—typical plot}$$

- Note: we do not require $m \cdot p_j \in$ integers , otherwise the problem is very difficult.
- Define the following function:

$$\phi(x, \xi) \stackrel{\text{def}}{=} \left[v^T(x)\, M^{-1}(\xi)\, v(x) - d(x, x_0)\right]$$

- Its first term is the prediction variance in estimating our scalar field at x, when ξ is used. $\phi = \infty$, if $\mathrm{Det}[M(\xi)] = 0$.

The main result

Theorem (optimality conditions):

- Assume that $d(x, x_0)$ is continuous in X as a function of x. X is a compact set.
- There exists ξ such that $\mathrm{Det}[M(\xi)] > 0$.
- ξ^* is optimal ($\max_\xi F(\xi)$) if and only if

$$\max_{x \in X} \phi(x, \xi^*) = r - D(\xi^*), \qquad (7)$$

  where $r = \dim(a)$ – the number of param.
- Condition (7) is the base for constructing the algorithm for searching ξ^*, by modifying the Wynn-Fedorov method.

Search for the optimal configuration

Step 0 Select ξ_0 with $\mathrm{Det}[M(\xi_0)] > 0$, accuracy $\epsilon > 0$ and set $k = 0$.

Step 1 find $\eta_k = \arg\max_{x \in X} \phi(x, \xi_k)$, if $\phi(\eta_k, \xi_k) \geq r - D(\xi_k)$ – go to 2,

Step 2 If $\phi(\eta_k, \xi_k) - (r - D(\xi_k)) < \epsilon$, then stop, otherwise, go to Step 3.

Step 3 Select $0 < \alpha_k < 1$, multiply all p_j's in ξ_k by $(1 - \alpha_k)$. Add η_k as a new sensor's position and attach weight α_k to it. Rename this as ξ_{k+1}, $k := k + 1$, go to Step 1.

Searching 2

- We omit the proof of convergence.
- In Step 1 task: $\eta_k = \arg\max_{x \in X} \phi(x, \xi_k)$ is difficult ($\phi(x, \xi_k)$) is multi-modal), but it is not necessary to find the global maximum. It suffices to find $\tilde{\eta}_k$ such that $\phi(\tilde{\eta}_k, \xi_k) > r - D(\xi_k)$.
- To this end: SQP with filter, evolutionary search or selective random search (with the normalized $\phi(x, \xi_k)$ as the pd.f. of search).

Searching 3

- In Step 3 one can select α_k in a number of ways: as a predefined sequence such that $\sum_{k=1}^{\infty} \alpha_k$ is divergent ($\alpha_k = 1/(1+k)$) or as a local minimizer of ϕ or by optimizing all the weights in a spirit of Fellman and Torsney procedure.

Searching 4

We shall describe the last possibility in more details. Let p_i^0, $i = 1, 2, \ldots, m$ denotes weights of a current design ξ_k. Then the weights optimization runs as follows:

$$p_i^{l+1} = p_i^l \cdot \frac{\phi(x_i, \xi_k^l)}{r - D(\xi_k^l)}, \tag{8}$$

until weights do not differ too much. Above x_i's are the support points of ξ_k, while ξ_k^l is a design with the same support, but with weights arising from (8).

Examples – conventions

- We symmetrize sensors' allocation
- When the plot X is rotationally symmetric so as ϕ function, then the symmetrization does not spoil the optimality of the design.
- The rationale behind the symmetrization – tilling larger regions by sticking together smaller domains.
- Sensors at the borders should send a part of their measurements to different sinks.
- A generic sub-field is a unit disc.

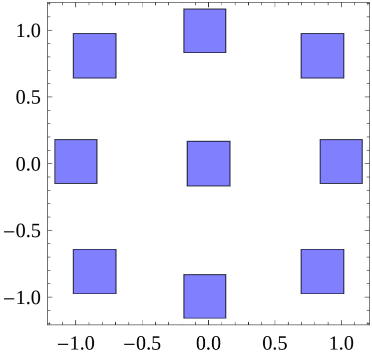
Examples – conventions 2

- Circles (or squares) are placed at points where sensors should be placed.
- Their areas are proportional to the frequencies of their activations.
- In all the examples we have used the same, intentionally badly selected, starting design:

$$\begin{bmatrix} \{+/-0.5, +/-0.5\} & 0.11 \\ \{0.75, 1.\} & 0.11 \\ \{-1., 0.\} & 0.11 \\ \{1., 0.75\} & 0.11 \\ \{0.75, -1.\} & 0.11 \\ \{0.75, 0.75\} & 0.11 \end{bmatrix} \quad (9)$$

Example 1

Penalty $d(x,t) = 2.0 * (x^2 + t^2 - 0.25)^2$, linear model with the interaction of spatial variables: $v(x,t) = [1, x, t, x\,t]$. Optimal sensors' allocation:



All weights = 0.11.

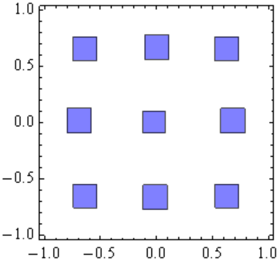
Example 2

The trigonometric base:

$$v(x) = [1,\ \sin(\pi\, x^{(1)}), \cos(\pi\, x^{(1)})] \otimes [1,\ \sin(\pi\, x^{(2)}), \cos(\pi\, x^{(2)})],$$

where $\otimes$ is the Kronecker product. The penalty:

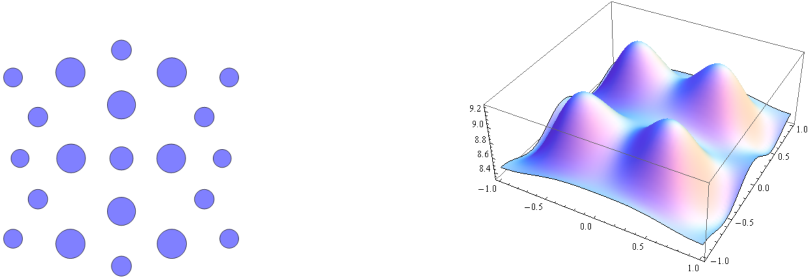
$$d(x, t) = 5.0 * (x^2 + t^2 - 0.5)^2$$



In opposite to D-optimal design, frequencies of activating sensors are different in different locations.

Example 3

The same base as above, but with penalty function: $d(x,t) = 1.5 * \mathrm{Exp}[-4.0 * (x^2 + t^2)]$.



Left panel – optimal allocation (nice tilling possible), right panel – ϕ surface for optimal allocation.

Conclusions

- The extension of the optimal experiment design theory is proposed.
- It allows to incorporate features, which are specific for WSN's, e.g., the penalty for excessive power consumption, leading to a balance between the estimation accuracy and the consumption for transmission.
- The modified version of the Wynn-Fedorov algorithm + many modifications (SQP with filter or selective random search+ weights optimization by the modified Torsney-Fellman algorithm) occurred to be efficient.

Conclusions 2

- The optimal allocations of sensors and the frequencies of their activations are such that they are easily repeatable in larger areas.
- The approach was presented as if the spatial field to be estimated is static (constant in time), but for one series of observations it is not reasonable to put so many effort for sensors' allocation. In fact, the approach is useful for quasi-stationary random fields, when the mean changes slowly in time (e.g., environmental pollution).

Kitchen – outline of the proof

Lemma For any sensing design $\xi \in \Xi(X)$ with nonsingular $M(\xi)$ we have (easy to prove):

$$r - D(\xi) \leq \max_{x \in X} \phi(x, \xi). \tag{10}$$

For ξ^* the equality in (10) holds. Indeed, $\forall\, \xi'$

$$\frac{d}{d\,\alpha} F((1-\alpha)\,\xi^* + \alpha\,\xi')\Big|_{\alpha=0+} \leq 0 \tag{11}$$

implies the reverse inequality. The proof of sufficiency is more subtle (uses the strict concavity of $F(\xi)$).

Selected bibliography

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