An Averaged AC Models Accuracy Evaluation of Non-Isolated Matrix-Reactance PWM AC Line Conditioners

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Acknowledgement

The Polish State Committee for Scientific Research, Project KBN No 8 T10A 028 16, supported this work.

Keywords

«Converter circuits, Power conditioning, Modelling»

Abstract

This paper deals with matrix-reactance PWM AC line conditioners (MRACLCs). Theoretical preliminary to averaged modelling of these conditioners with current time averaging operator and averaging errors estimation for fundamental harmonics of state variable is presented. On the base of theoretical consideration there is steady-state averaged AC models quantitative accuracy evaluation formulated in this paper. As an application of the theory, examples of the analysis of averaging errors for conditioners based on buck-boost and Sepic topologies are presented. In this paper one can find averaging errors analysis both as a function of switching frequency as well as a function of load resistance.

Introduction

In industrial practice AC line conditioners with thyristor power controllers are commonly used. Major disadvantages of those are: generation of higher harmonics in the source current, generation of displacement power at phase angle control and generation of subharmonics at integral control. In order to eliminate these unfavourable properties the matrix or matrix-reactance PWM AC line conditioners (MACLCs, MRACLCs) are used [1-8]. In MRACLCs well-known DC/DC converter topologies are applied. Some of them have capability to buck-boost AC voltage transformation without an electromagnetic transformer [5-8]. The utilisation of their interesting properties is continuously developed. They are also used for frequency conversion with buck-boost AC voltage transformation [9], to direct buck-boost AC/DC conversion [10], as a controlled reactance to generate/absorb of reactive power [11] or as an additional conditioner to compensate of voltage sags in electric power distribution systems [12].
The basic energetic properties analysis of MRACLCs is complex due to their switching behaviour. There are either an exact or approximated methods of analysis these conditioners, respectively as in [9, 13] or in [5-8, 14]. Similarly to DC/DC converters analysis, the averaged models obtained by averaged state-space method [15, 16] are commonly applied to steady-state basic energetic properties analysis, which is valid for variable fundamental harmonics. In works [17-19] extension of averaging theory for DC/DC converters is presented. It allows to ripple (averaging error) estimation but one cannot be adapted to averaging errors estimation when an AC input as in presented conditioners is used. It comes from type of averaging operator that is used.
This paper discusses the MRACLCs averaged models accuracy. There is a brief description of typical two-switch MRACLC topologies and theoretical preliminary of averaging with current time averaging operator as in [20], which is located at the beginning of the paper. Next, on the base of theoretical

consideration errors estimation of steady-state averaged AC models is formulated. Furthermore examples of the analysis of averaging errors for conditioners based on buck-boost and Sepic topologies are presented. In this paper one can also find averaging errors analysis both as a function of switching frequency as well as a function of load resistance.
The main purpose of this paper is quantitative evaluation of MRACLCs averaged models accuracy that is necessary to their effective introduction over investigations of presented conditioner applications. The conclusions follow in the last section.

Description of typical non-isolated MRACLC topologies

Typical single-phase two-switch MRACLC topologies are similar to well-known DC/DC converters and can be divided on buck and boost families [14-16]. In fig.1 single-phase two-switch circuit models of buck family MRACLC (Fig.1a, c, e, buck, buck-boost and Zeta topologies) and boost one (Fig.1b, d, f, boost, Ćuk and Sepic topologies) are shown. In these conditioners, bi-directional switches must be used and it is the main difference in comparison to DC/DC converter topologies. There are detailed operation descriptions of these conditioners in works [5-9].

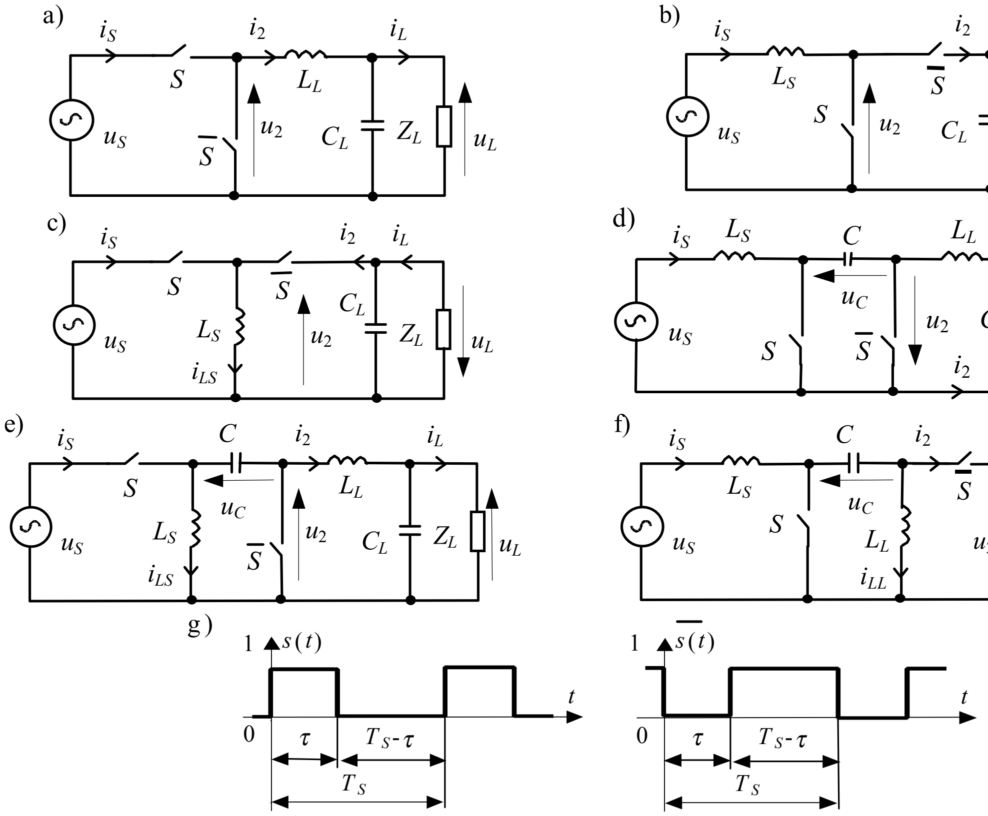

Fig.1: Typical single-phase two-switch MRACLC topologies, a) buck, b) boost, c) buck-boost, d) Ćuk, e) Zeta, f) Sepic, g) time waveforms of switch control signals

Theoretical preliminary

The state-space method analysis

For each MRACLCs shown in fig.1 there are two circuit states (first state when switch S is on and switch $\overline{S}$ is off and second state when both switches are in inverse states). The state-space description can be written as:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}_1\boldsymbol{x} + \boldsymbol{B}_1\boldsymbol{u}, \quad \boldsymbol{y} = \boldsymbol{C}_1\boldsymbol{x} \quad \text{for } 0 \le t < dT_S \\ \dot{\boldsymbol{x}} &= \boldsymbol{A}_2\boldsymbol{x} + \boldsymbol{B}_2\boldsymbol{u}, \quad \boldsymbol{y} = \boldsymbol{C}_2\boldsymbol{x} \quad \text{for } dT_S \le t < T_S \end{aligned} \tag{1}$$

where: $d = \tau / T_S$ – pulse duty ratio of switch control signal satisfying $0 \le d \le 1$.

A circuit states periodical changes and (1) can be expressed by:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}(t)\boldsymbol{x} + \boldsymbol{B}(t)\boldsymbol{u} = \boldsymbol{F}(x,t) \\ \boldsymbol{y} &= \boldsymbol{C}(t)\boldsymbol{x} \end{aligned} \tag{2}$$

where: $\boldsymbol{A}(t) = \boldsymbol{A}_1\, s(t) + \boldsymbol{A}_2\, [1 - s(t)]$, $\boldsymbol{B}(t) = \boldsymbol{B}_1\, s(t) + \boldsymbol{B}_2\, [1 - s(t)]$, $\boldsymbol{C}(t) = \boldsymbol{C}_1\, s(t) + \boldsymbol{C}_2\, [1 - s(t)]$, $\boldsymbol{F}(x, t)$ – continuous function of x and discontinuous of t.

Solution of equation (1) is expressed as [21]:

$$\boldsymbol{x}(t) = e^{\boldsymbol{A}_i\left(t - t_k^i\right)}\boldsymbol{x}\left(t_k^i\right) + \int_{t_k^i}^{t} e^{\boldsymbol{A}_i(t-\vartheta)}\boldsymbol{B}_i(\vartheta)u_S(\vartheta)d\vartheta \tag{3}$$

where: i = 1, 2 – number of time intervals during switching period T_S; t_k^i - initial time of i interval during k switching period; $t_k^i = \begin{cases} t_k, i = 1 \\ t_k + \tau, i = 2 \end{cases}$; t_k – initial time of k switching period; $1 \le k \le N$; $N = T/T_S$; $t_1 = 0$; $t_2 = T_S$, … $t_N = T - T_S$; $u_S(t) = U_{\max}\sin\omega t$; U_{max} – amplitude of supplying voltage; $\boldsymbol{A}_i(t) = \begin{cases} \boldsymbol{A}_1, t_k \le t \le t_k + \tau, (s = 1), \\ \boldsymbol{A}_2, t_k + \tau \le t \le t_k + T_S, (s = 0) \end{cases}$; $\boldsymbol{B}_i(t) = \begin{cases} \boldsymbol{B}_1, t_k \le t \le t_k + \tau, (s = 1), \\ \boldsymbol{B}_2, t_k + \tau \le t \le t_k + T_S, (s = 0) \end{cases}$.

The second component of (3) is given as:

$$\int_{t_k^i}^{t} e^{\boldsymbol{A}_i(t-\vartheta)}\boldsymbol{B}_i(\vartheta)u_S(\vartheta)d\vartheta = e^{\boldsymbol{A}_i(t-t_k)}\boldsymbol{K}_i\left(t_k^i\right) - \boldsymbol{K}_i(t) \tag{4}$$

where: $\boldsymbol{K}_i(t) = \left(\boldsymbol{A}_i^2 + \mathbf{1}\omega^2\right)^{-1}(\mathbf{1}\omega\cos\omega t + \boldsymbol{A}_i\sin\omega t)\boldsymbol{B}_i$; $\mathbf{1}$ – unit matrix; $(\ldots)^{-1}$- inverse matrix.
Initial condition solutions for first time interval during first switching period can be expressed as:

$$\boldsymbol{x}(0) = \boldsymbol{x}(T) = \left(\mathbf{1} - \prod_{k=1}^{N}\boldsymbol{H}\right)^{-1}\sum_{k=1}^{N}\left(\prod_{j=k+1}^{N}\boldsymbol{H}\right)\boldsymbol{P}_k \tag{5}$$

where: $\boldsymbol{H} = e^{\boldsymbol{A}_2(T_S-\tau)}e^{\boldsymbol{A}_1\tau}$; $\boldsymbol{P}_k = e^{\boldsymbol{A}_2(T_S-\tau)}\boldsymbol{G}_1(t_k) + \boldsymbol{G}_2(t_k+\tau)$; $\boldsymbol{G}_1(t_k) = e^{\boldsymbol{A}_1\tau}\boldsymbol{K}_1(t_k) - \boldsymbol{K}_1(t_k+\tau)$; $\boldsymbol{G}_2(t_k+\tau) = e^{\boldsymbol{A}_2(T_S-\tau)}\boldsymbol{K}_2(t_k+\tau) - \boldsymbol{K}_2(t_k+T_S)$.

Next initial conditions are calculated similarly by recursive using of (3).
Fundamental harmonics of Fourier series of the respective state variable are given as:

$$\boldsymbol{x}_1(t) = \boldsymbol{c}_{1x}\sin(\omega t + \boldsymbol{\psi}_{1x}) \tag{6}$$

where: $\boldsymbol{c}_{1x} = \sqrt{\boldsymbol{a}_{1x} + \boldsymbol{b}_{1x}}$; $\boldsymbol{a}_{1x}$, $\boldsymbol{b}_{1x}$ – Fourier series coefficients for fundamental harmonic of state variable which are described in appendix by (A1) and (A2); $\boldsymbol{\psi}_{1x}$ = arctg $\boldsymbol{a}_{1x}$ / $\boldsymbol{b}_{1x}$ – fundamental harmonic phase of state variable.

The averaged state-space method analysis

The averaged method, used for DC/DC converter analysis and introduced by Middlebrock and Ćuk [15], consisted in the time average of the right-hand side (RHS) time-varying equations (2) that is defined as [17, 19]:

$$G(x) = \lim_{T\to\infty}\frac{1}{T}\int_0^T F(x,t)dt \tag{7}$$

Because of $F(x, t)$ is periodic function, (7) can be written as:

$$G(x) = \frac{1}{T}\int_0^T F(x,t)dt \tag{8}$$

where: T - period of $F(x, t)$.

The averaged RHS of (2), $G(x)$, gives rise to a new time-invariable system of equations:

$$\begin{aligned}\dot{\bar{\boldsymbol{x}}} &= \boldsymbol{A}(d)\bar{\boldsymbol{x}} + \boldsymbol{B}(d)\bar{\boldsymbol{u}} \\ \bar{\boldsymbol{y}} &= \boldsymbol{C}(d)\bar{\boldsymbol{x}}\end{aligned} \tag{9}$$

where: $\boldsymbol{A}(d) \equiv \boldsymbol{A}_1 d + \boldsymbol{A}_2(1\text{-}d)$, $\boldsymbol{B}(d) \equiv \boldsymbol{B}_1 d + \boldsymbol{B}_2(1\text{-}d)$, $\boldsymbol{C}(d) \equiv \boldsymbol{C}_1 d + \boldsymbol{C}_2(1\text{-}d)$.

It should be noted that another averaging operator is needed to present AC/AC converters because of two periods (switching period T_S and supplying voltage period T) that occur in this case. Therefore following averaging operator is applied [18]:

$$\widetilde{G}(x) = \frac{1}{T_S}\int_{t-T_S}^{t} F(x,t)dt \tag{10}$$

where: T_S – period of switching function $s(t)$.

Then, the averaged RHS of (2), $\widetilde{G}(x)$, gives in steady state (when $d = \tau/T_S = D$) following averaged equation:

$$\dot{\bar{\boldsymbol{x}}} = \boldsymbol{A}(D)\bar{\boldsymbol{x}} + \frac{1}{T_S}\int_{t-T_S}^{t} \boldsymbol{B}(\boldsymbol{t})U_{\max}\sin\omega t dt \tag{11}$$

For boost family of presented MRACLCs (fig.1b, d, f) is:

$$\frac{1}{T_S}\int_{t-T_S}^{t} \boldsymbol{B}U_{\max}\sin\omega t dt = \boldsymbol{B}\frac{1}{T_S}\int_{t-T_S}^{t} U_{\max}\sin\omega t dt = \boldsymbol{B}U_{\max}\frac{N\sin\frac{\pi}{N}\sin\left(\omega t - \frac{\pi}{N}\right)}{\pi} \tag{12}$$

Whereas, for buck family of ones (fig.1a, c, e) is:

$$\frac{1}{T_S}\int_{t-T_S}^{t} \boldsymbol{B}(\boldsymbol{t})U_{\max}\sin\omega t dt \approx \boldsymbol{B}D\frac{1}{T_S}\int_{t-T_S}^{t} U_{\max}\sin\omega t dt = \boldsymbol{B}DU_{\max}\frac{N\sin\frac{\pi}{N}\sin\left(\omega t - \frac{\pi}{N}\right)}{\pi} \tag{13}$$

Hence, equation (11) can be written as:

$$\dot{\bar{\boldsymbol{x}}} = \boldsymbol{A}(D)\bar{\boldsymbol{x}} + \boldsymbol{B}\sin\left(\omega t - \frac{\pi}{N}\right)M \tag{14}$$

where: $M = U_{\max}\left(N\sin\frac{\pi}{N}\right)\frac{1}{\pi}$ for MRACLCs belonging to boost family and $M = DU_{\max}\left(N\sin\frac{\pi}{N}\right)\frac{1}{\pi}$ for MRACLCs belonging to buck one.

Thus, the averaged solution of (1) can be expressed as:

$$\bar{\boldsymbol{x}}(t) = \left[\boldsymbol{A}^2(D) + \mathbf{1}\omega^2\right]^{-1}\left[-\boldsymbol{A}(D)\sin\left(\omega t - \frac{\pi}{N}\right) - \mathbf{1}\omega\cos\left(\omega t - \frac{\pi}{N}\right)\right]\boldsymbol{B}M \tag{15}$$

Consider that in steady state averaged model (10) is linear so fundamental harmonics of Fourier series of the respective averaged state variables are given as:

$$\bar{\boldsymbol{x}}_1(t) = \bar{\boldsymbol{x}}(t) = \bar{\boldsymbol{c}}_{1x}\sin(\omega t + \bar{\boldsymbol{\psi}}_{1x}) \tag{16}$$

where: $\bar{\boldsymbol{c}}_{1x} = \sqrt{\bar{\boldsymbol{a}}_{1x} + \bar{\boldsymbol{b}}_{1x}}$; $\bar{\boldsymbol{a}}_{1x}, \bar{\boldsymbol{b}}_{1x}$ – Fourier series coefficients for fundamental harmonic of averaged state variable which are described in appendix by (A3) and (A4); $\bar{\boldsymbol{\psi}}_{1x} = \operatorname{arctg} \bar{\boldsymbol{a}}_{1x} / \bar{\boldsymbol{b}}_{1x}$ – fundamental harmonic phase of averaged state variable.

Averaging errors for fundamental harmonic of state variables

Taking into consideration averaged solution (15) for switching frequency $f_S = 1/T_S$ increased to infinity, we obtain:

$$\tilde{\boldsymbol{x}}(t) = \lim_{N\to\infty} \bar{\boldsymbol{x}} = U_{\max}\left[\boldsymbol{A}^2(D) + \mathbf{1}\omega^2\right]^{-1}\left[-A(D)\sin\omega t - \mathbf{1}\omega\cos\omega t\right]\boldsymbol{B}M \tag{17}$$

where: $M = U_{max}$ for MRACLCs belonging to boost family and $M = DU_{max}$ for buck one.

From comparison both original system solution (3) expressed by (6) and averaged solution (17) results that in steady state only for infinitely small switching period T_S both a state-space and an averaged solutions of (1) for fundamental harmonic are the same, i. e.:

$$\tilde{\boldsymbol{x}}(t) = \lim_{N\to\infty} \boldsymbol{x}_1(t) \tag{18}$$

For finite T_S, the averaged solution of the state variable is burden with error. There are amplitude and phase errors in averaged solutions, which are perform in general form by following expressions:

$$\delta_{xa} = \frac{\left|c_{1x} - \bar{c}_x\right|}{c_{1x}} \tag{19}$$

$$\delta_{x\psi} = \psi_{1x} - \bar{\psi}_x \tag{20}$$

Theoretical and simulation test results

Schematic diagrams of matrix-reactance PWM AC line conditioners, which have been investigated, are shown in fig.1c (buck-boost topology) and in fig.1f (Sepic topology). Ones relevant circuit parameters and state-space matrixes or vectors are collected in appendix tables AI, AII and described by appendix expressions (A3) and (A4) respectively. Presented investigations have been carried out at assumption (21), i.e. with load matching.

$$L_S = L_L = L; C = C_L = C \text{ and } R_L = \sqrt{L_S / C} = \sqrt{L_L / C_L} = \sqrt{L / C} \tag{21}$$

Load voltage

Exemplary simulation time waveforms of source u_S and load u_L voltages in original system of buck-boost and Sepic conditioners, obtained by means of PSpice, are shown in fig.2.

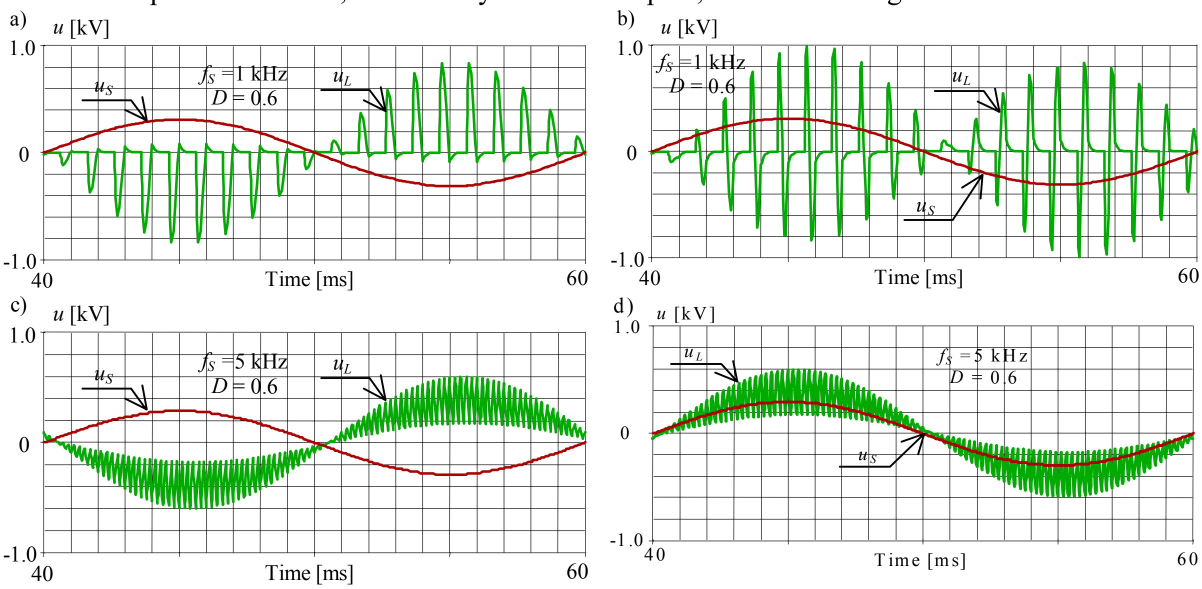

Fig.2: Simulation time waveforms of source u_S and load u_L voltages,
a), c) for buck-boost conditioner, b), d) for Sepic conditioner

The calculation results of load voltage amplitude and phase averaging errors as a function of impulse duty factor D and switching frequency f_S, at fixed load resistance R_L for MRACLCs based on buck-boost and Sepic topologies, are shown in fig.3. These results confirmed that for presented conditioners both amplitude and phase load voltage averaging errors decreasing together with switching frequency increasing. Furthermore for 5 kHz switching frequency i.e. about $3/\left(2\pi\sqrt{LC}\right)$ the amplitude errors are smaller then approximately 20 %, whereas phase errors are smaller then 0.1 rad.

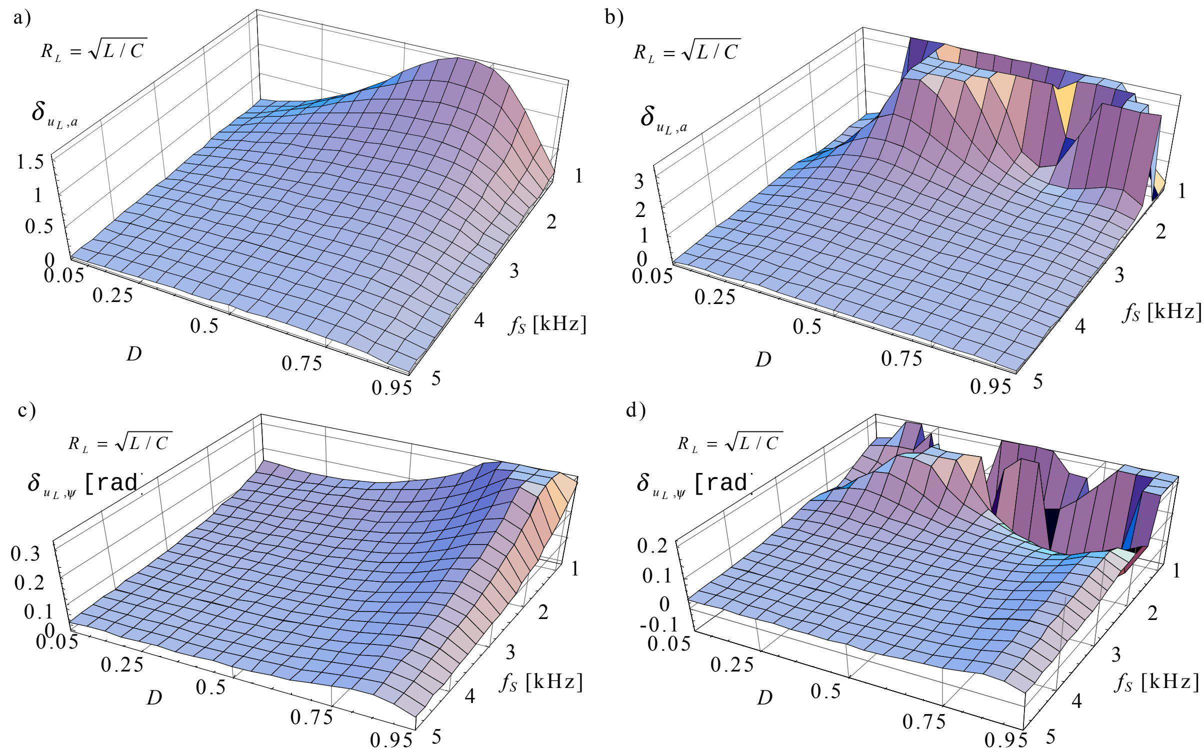

Fig.3: Amplitude and phase load voltage averaging errors versus of impulse duty factor D and switching frequency f_S, at fixed load resistance R_L, a), c) for buck-boost condit., b), d) for Sepic one

The calculation results of load voltage amplitude and phase averaging errors as a function of impulse duty factor D and load resistance R_L at fixed switching frequency f_S, for presented conditioners, are shown in fig.4.

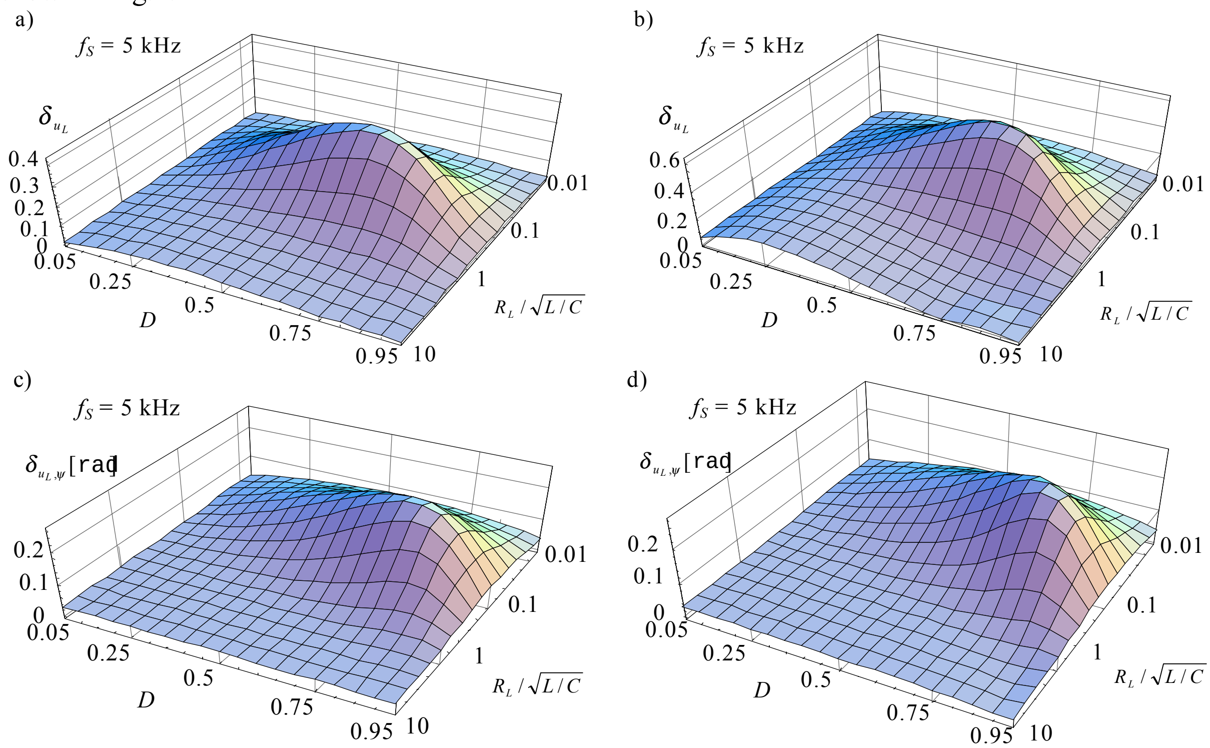

Fig.4: Amplitude and phase load voltage averaging errors versus of impulse duty factor D and load resistance R_L at fixed switching frequency f_S, a), c) for buck-boost condit., b), d) for Sepic one

Referring to fig.4 it is visible that greatest amplitude and phase averaging errors of load voltage occur when $0.01 < R_L / \sqrt{L/C} < 1$. For $R_L / \sqrt{L/C} > 1$ the amplitude error of load voltage for Sepic conditioner is slightly greater then for buck-boost conditioner.

Source current

Exemplary simulation time waveforms of source currents i_S in original system of buck-boost and Sepic conditioners, obtained by means of PSpice, are shown in fig.5.

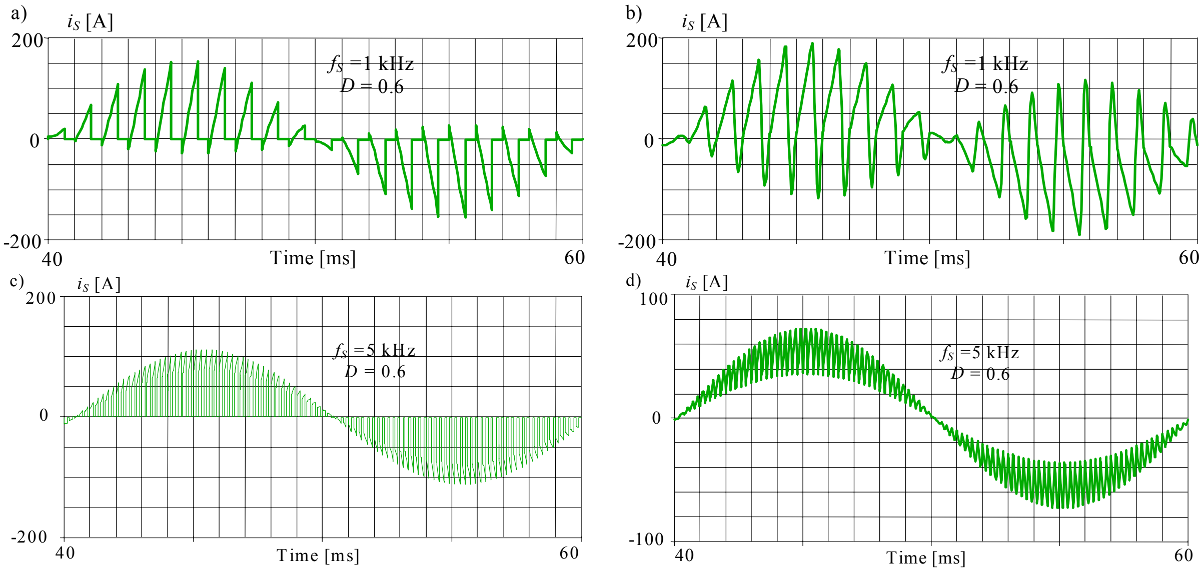

Fig.5: Simulation time waveforms of source currents i_S,
a), c) for buck-boost conditioner, b), d) for Sepic conditioner

The calculation results of amplitude and phase averaging errors of source current as a function of impulse duty factor D and switching frequency f_S, at fixed load resistance R_L for MRACLCs based on buck-boost and Sepic topologies, are shown in fig.6.

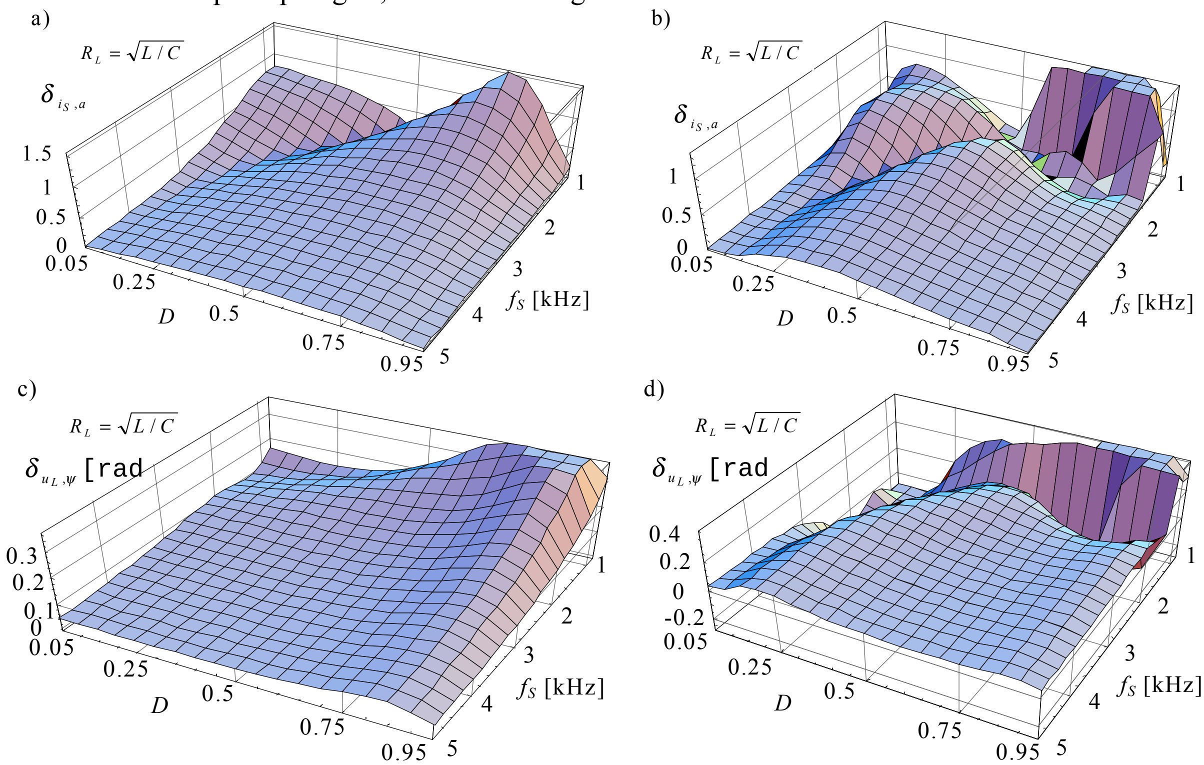

Fig.6: Amplitude and phase source current averaging errors versus of impulse duty factor D and switching frequency f_S, at fixed load resistance R_L, a), c) for buck-boost condit., b), d) for Sepic one

These results confirmed that similarly as for load voltage, both amplitude and phase source current averaging errors decreasing together with switching frequency increasing. In this case, for 5 kHz switching frequency i.e. about $3/\left(2\pi\sqrt{LC}\right)$ the amplitude errors are also smaller then approximately 20 %, whereas phase errors are smaller then 0.1 rad.

Next, calculation results of source current amplitude and phase averaging errors as a function of impulse duty factor D and load resistance R_L, at fixed switching frequency f_S, for presented conditioners, are shown in fig.7. These results can be commented similarly as for load voltage (fig.4), moreover for buck-boost one (fig.7a, c) increasing of both amplitude and phase errors when $R_L/\sqrt{L/C}>1$ are visible.

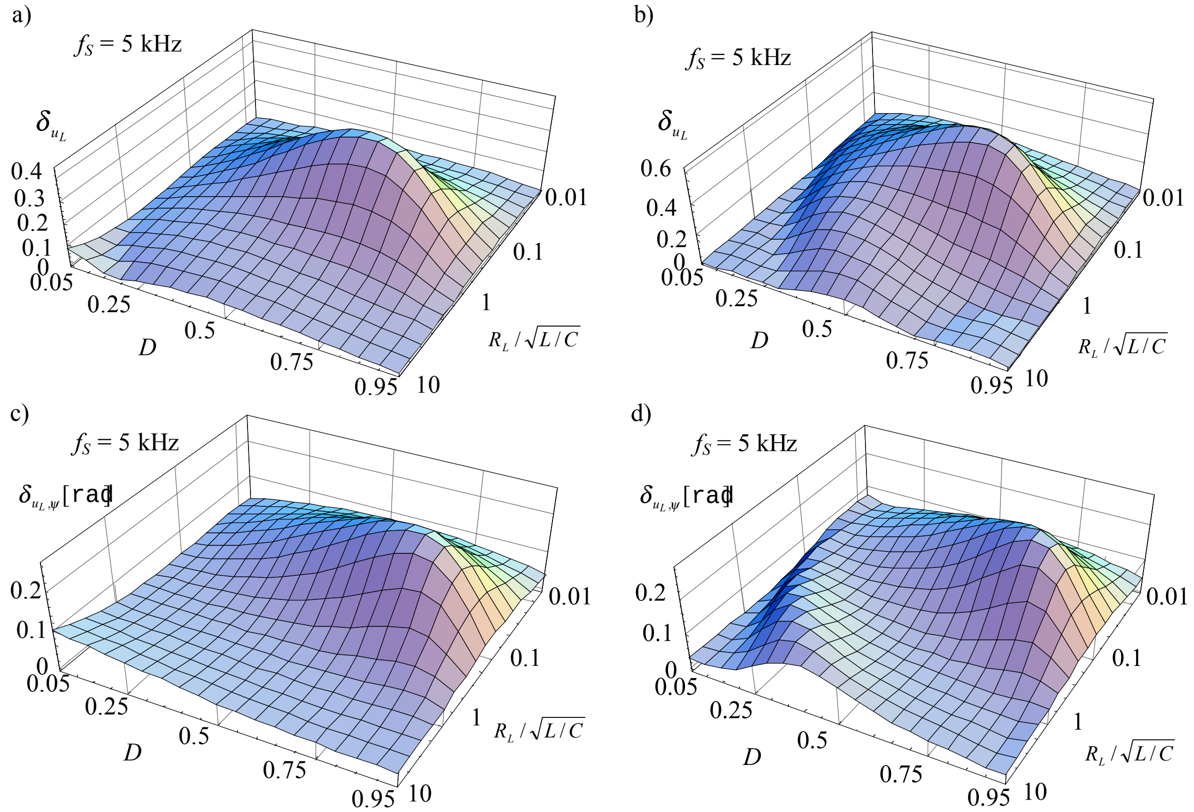

Fig.7: Amplitude and phase load voltage averaging errors versus of impulse duty factor D and load resistance R_L at fixed switching frequency f_S, a), c) for buck-boost condit., b), d) for Sepic one

Displacement factor

It is obvious that averaging errors have influence on errors of energetic properties described by means of averaged variable, which are obtained for presented MRACLCs. Exemplary results of source current phase averaging errors influence on averaging error of displacement factor (DF) for buck-boost and Sepic conditioners are depicted in fig.8. The averaging error of DF is calculated as:

$$\delta_{\cos\psi_{1,i_S}} = \cos\psi_{1,i_S} - \cos\overline{\psi}_{1,i_S} \quad (22)$$

where: $\cos\psi_{1,i_S}, \cos\overline{\psi}_{1,i_S}$ - displacement factor for original and averaged solution respectively.

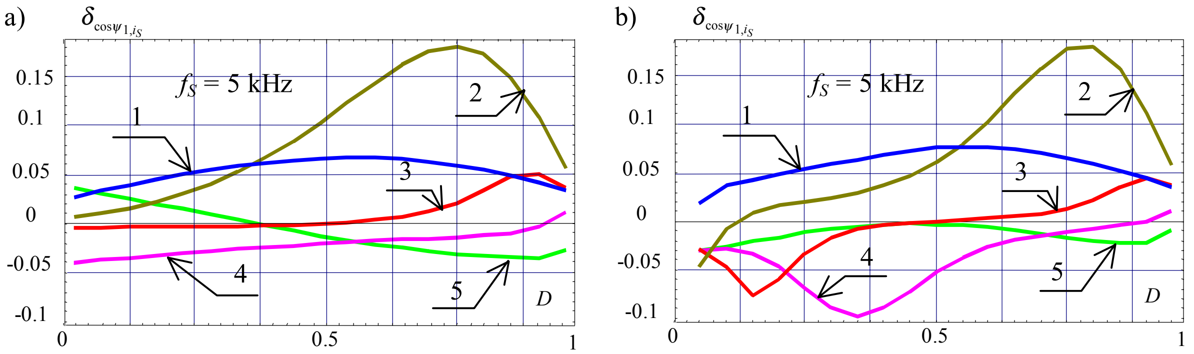

Fig.8: Averaging errors of displacement factor versus impulse duty factor D for $R_L = l\times\sqrt{L_L/C_L}$,
a) for buck-boost conditioner, b) for Sepic one
1 – l =0.01, 2 – l = 0.1, 3 - l = 1, 4 – l = 10, 5 – l = 100

Conclusions

In this paper theoretical analysis of matrix-reactance PWM AC line conditioners based on state-space and averaged state-space methods has been performed. The averaging with current time averaging operator has been applied. From comparison both original and averaged system solutions results, that in steady state, only for infinitely small switching period T_S both a state-space and an averaged solutions for fundamental harmonic are the same. For finite T_S, the averaged solution of the state variable is burden with error. Analytical expressions for calculation of averaging errors for fundamental harmonic have been formulated. On this base, quantitative accuracy evaluation of non-isolated MRACLCs averaged AC models has been presented, which has covered averaging errors analysis both as a function of switching frequency as well as a function of load resistance.

Obtained results confirm that for presented conditioners both amplitude and phase averaging errors of state variables decreasing along with switching frequency increasing. Furthermore for 5 kHz switching frequency i.e. about $3/\left(2\pi\sqrt{LC}\right)$ the amplitude errors are smaller then approximately 20 %, whereas phase errors are smaller then 0.1 rad. Greatest amplitude and phase averaging errors of state variables occur when $0.01 < R_L/\sqrt{L/C} < 1$. Further work considering averaging errors evaluation of all energetic properties of presented conditioners is under progress. This study should be considered as a first approach to aid in averaged AC models accuracy evaluation these conditioner systems.

Appendix

Fourier series coefficients of respective state variable for fundamental harmonics have the following forms:

$$\begin{aligned}\boldsymbol{a}_{1x} = \frac{2}{T}\sum_{k=1}^{N}\sum_{i=1}^{2}\left(\boldsymbol{A}_i^2 + \mathbf{1}\omega^2\right)^{-1}\Bigg\{\Big\{\boldsymbol{A}_i\left[e^{\boldsymbol{A}_i\left(t_k^{\bullet i}-t_k^i\right)}\cos\omega t_k^{\bullet i} - \mathbf{1}\cos\omega t_k^i\right] + \omega\left[e^{\boldsymbol{A}_i\left(t_k^{\bullet i}-t_k^i\right)}\sin\omega t_k^{\bullet i} - \mathbf{1}\sin\omega t_k^i\right]\Big\}\bullet \\ \bullet\left\{\boldsymbol{x}\left(t_k^i\right) + \boldsymbol{K}_i\left(t_k^i\right)\right\} - \boldsymbol{B}_i\frac{\omega}{2}t_k^{*i} + \boldsymbol{A}_i\boldsymbol{B}_i\left(\cos 2\omega t_k^{*i} - \cos 2\omega t_k^i\right)\frac{1}{4\omega} - \boldsymbol{B}_i\left(\sin 2\omega t_k^{*i} - \sin 2\omega t_k^i\right)\frac{1}{4}\Bigg\}\end{aligned} \tag{A1}$$

$$\begin{aligned}\boldsymbol{b}_{1x} = \frac{2}{T}\sum_{k=1}^{N}\sum_{i=1}^{2}\left(\boldsymbol{A}_i^2 + \mathbf{1}\omega^2\right)^{-1}\Bigg\{\Big\{\boldsymbol{A}_i\left[e^{\boldsymbol{A}_i\left(t_k^{\bullet i}-t_k^i\right)}\sin\omega t_k^{\bullet i} - \mathbf{1}\sin\omega t_k^i\right] + \omega\left[-e^{\boldsymbol{A}_i\left(t_k^{\bullet i}-t_k^i\right)}\cos\omega t_k^{\bullet i} - \mathbf{1}\cos\omega t_k^i\right]\Big\}\bullet \\ \bullet\left\{\boldsymbol{x}\left(t_k^i\right) + \boldsymbol{K}_i\left(t_k^i\right)\right\} - \boldsymbol{A}_i\boldsymbol{B}_i\frac{1}{2} + \boldsymbol{A}_i\boldsymbol{B}_i\left(\sin 2\omega t_k^{*i} - \sin 2\omega t_k^i\right)\frac{1}{4\omega} + \boldsymbol{B}_i\left(\cos 2\omega t_k^{*i} - \cos 2\omega t_k^i\right)\frac{1}{4}\Bigg\}\end{aligned} \tag{A2}$$

where: t_k^{*i} - successive time after t_k^i.

Fourier series coefficients of respective averaged state variable for fundamental harmonics have the following forms:

$$\boldsymbol{a}_{1x} = \left[\boldsymbol{A}^2(D) + \mathbf{1}\omega^2\right]^{-1}\left[\boldsymbol{A}(D)\sin\frac{\pi}{N} - \mathbf{1}\omega\cos\frac{\pi}{N}\right]\boldsymbol{B}(D)M \tag{A3}$$

$$\boldsymbol{b}_{1x} = \left[\boldsymbol{A}^2(D) + \mathbf{1}\omega^2\right]^{-1}\left[-\boldsymbol{A}(D)\cos\frac{\pi}{N} - \mathbf{1}\omega\sin\frac{\pi}{N}\right]\boldsymbol{B}(D)M \tag{A4}$$

Table AI. Calculation and simulation test circuit parameters

Name of parameter	Symbol	Value
Phase supply voltage	U_{max}	310 V / 50 Hz
Phase load impedance	Z_L	$Z_L = R_L = 10\ \Omega$
Switching frequency	f_S	5 kHz
Source inductance	L_S	1 mH
Buffer capacitance	C	10 μF
Load inductance	L_L	1 mH
Load capacitance	C_L	10 μF
Parasitic resistance	r	0.1 Ω

Table AII. State-space A and input B matrixes

Buck-boost topology	$\boldsymbol{x}=\begin{bmatrix} i_{LS} \\ u_L \end{bmatrix}\ \boldsymbol{A}_1=\begin{bmatrix} \frac{-r}{L_S} & 0 \\ 0 & \frac{-1}{R_L C_L} \end{bmatrix};\ \boldsymbol{A}_2=\begin{bmatrix} \frac{-r}{L_S} & \frac{-1}{L_S} \\ \frac{1}{C_L} & \frac{-1}{R_L C_L} \end{bmatrix};\ \boldsymbol{B}_1=\begin{bmatrix} \frac{1}{L_S} \\ 0 \end{bmatrix};\ \boldsymbol{B}_2=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
Sepic topology	$\boldsymbol{x}=\begin{bmatrix} i_S \\ i_2 \\ u_C \\ u_L \end{bmatrix};\ \boldsymbol{A}_1=\begin{bmatrix} \frac{-r}{L_S} & 0 & 0 & 0 \\ 0 & \frac{-r}{L_L} & \frac{-1}{L_L} & 0 \\ 0 & \frac{1}{C} & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{R_L C_L} \end{bmatrix};\ \boldsymbol{A}_2=\begin{bmatrix} \frac{-r}{L_S} & 0 & \frac{-1}{L_S} & \frac{-1}{L_S} \\ 0 & \frac{-r}{L_L} & 0 & \frac{1}{L_L} \\ \frac{1}{C} & 0 & 0 & 0 \\ \frac{1}{C_L} & \frac{-1}{C_L} & 0 & \frac{-1}{R_L C_L} \end{bmatrix};$ $\boldsymbol{B}_1=\begin{bmatrix} \frac{1}{L_S} \\ 0 \\ 0 \\ 0 \end{bmatrix};\ \boldsymbol{B}_2=\begin{bmatrix} \frac{1}{L_S} \\ 0 \\ 0 \\ 0 \end{bmatrix}$

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