Implementation of Multirate Modified Wave Digital Filters
Using TMS320C40 and TMS320C6000 Digital Signal Processors

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Abstract

This paper describes implementation of multirate modified lattice wave digital filters using modern
TMS320 family digital signal processors. Cascaded interpolator based on bireciprocal modified lattice
wave digital filters for interpolating of high quality audio signals is presented. The interpolator is
implemented in TMS320C40 digital signal processor. The methodology for this and the results are
presented. The frequency response of the cascaded interpolator realized with the TMS320C40
processor is analyzed for interpolation ratio \( R=8 \). This interpolator achieves the signal-to-noise and
distortion ratio \( S_{INAD} \) near to 90dB and the passband ripple of \( \delta_p \approx 8 \cdot 10^{-9} \) dB.

1. Introduction

Wave digital filters (WDF’s) are known to have many advantageous properties [4]. They have a
relatively low passband sensitivity to coefficients, small rounding errors, big resistivity to parasitic
oscillations (limit cycles), great dynamic range, low level of the rounding noise, etc. Especially
favorable are the lattice wave digital filters. They are well suited for the high quality audio signal
processing.

Fig. 1. Simplified block diagrams of lattice wave digital filters: (a), (b) analysis filter bank,
(c) synthesis filter bank

Lattice wave digital filters are built with two blocks realizing all-pass functions \( S_1(z) \) and \( S_2(z) \).
Typically blocks \( S_1(z) \) and \( S_2(z) \) are realized by a cascade of first-order and second-order all-pass
sections. The transfer function of a lattice WDF can be written as

\[
H(z) = 0.5(S_1(z) + S_2(z)) .
\] (1)

These allpass filters can be realized in several ways described in [4]. One approach that yields parallel
and modular filter algorithms is the use of cascaded first and second-order sections as shown in Fig. 2.
Block diagram of a typical, illustrative lattice wave digital filter is depicted in Fig. 2. The first and the
second-order all-pass sections are here realized using symmetric two-port adaptors. Typical classical
two-port adaptors are depicted in Fig. 3. Reflection signals \( b_1 \) and \( b_2 \) can be calculated by equations

\[
\begin{cases}
\gamma \gamma \\
\gamma 
\end{cases}
\] (2)

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Both classic first-order all-pass sections shown in Fig. 3b, c require a single multiplication and three additions. These two versions have, however, different lengths of critical paths. The critical path of the all-pass section in Fig. 3b consists of a single multiplier and two summers, while that in Fig. 3c —of a single multiplier and three summers. The first version is better for the implementation in a digital signal processor with a parallel instruction set.
Another section depicted in Fig. 4b requires three multiplications and two additions but its critical path consists of two multiplications and one addition only. This section is better for a DSP realization.

Fig. 4. Block diagrams of first-order all-pass sections with equal numbers of multiplications and additions: (a) section with long critical path, (b) alternative section with shorter critical path

2. Modified Wave Digital Filters

Today, modern digital signal processors are designed to be able to calculate multiplication together with addition (or more) in a single operational cycle. In result the classical two-port adaptor structure of Fig. 3b is ineffective for the DSP realization, especially for the floating point arithmetic. That is why modified structures have been proposed. The idea of modified wave digital filters, i.e., those with equal numbers of additions and multiplications and with short critical paths were proposed by Fettweis in [5]. In Figure 5 this idea consisting in inclusion of two additional complementary multipliers is illustrated.

Fig. 5. Block diagram of the first-order modified all-pass section: (a) idea of complementary multipliers, (b) realization of the two-port adaptor

Reflection signals \( b_1 \) and \( b_2 \) of the modified two-port adaptor (Fig. 6a) can be calculated by equations

\[
\begin{align*}
  \gamma_{n} & = \gamma_{11} a_1 + \gamma_{12} a_2 \\
  b_2 & = \gamma_{21} a_1 + \gamma_{22} a_2 ,
\end{align*}
\]

in which coefficients \( \gamma_i \) are given by equations

\[
\begin{align*}
  \gamma_{11} & = -\frac{k_{a1}}{k_{d1}} \\
  \gamma_{12} & = \frac{(1 + \gamma_i)k_{a1}}{k_{d1}} \\
  \gamma_{21} & = \frac{(1 - \gamma_i)k_{a1}}{k_{d1}} \\
  \gamma_{22} & = \gamma_i 
\end{align*}
\]

Three cases for the realization of modified two-port adaptors are possible. They are described by equations listed in Table 1 and depicted in Figs. 6b, 6c and 6d. Every realization needs four operations: two multiplications and two additions. In cases 1 and 3, the critical path consists of only two arithmetic operations.
Fig. 6. Block diagrams of first-order modified all-pass sections: (a), (b) case 1, (c) case 2, (d) case 3

Table 1. Equations for the first-order modified all-pass sections

<table>
<thead>
<tr>
<th>Case 1 for: $\gamma_{11}=1$, $\gamma_{21}=1$</th>
<th>Case 2 for: $\gamma_{11}=1$, $\gamma_{21}=1$</th>
<th>Case 3 for: $\gamma_{11}=1$, $\gamma_{21}=1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{a1} = 1/(1-\gamma)$, $k_{d1} = 1+\gamma$</td>
<td>$k_{a1} = -(1+\gamma)/\gamma$, $k_{d1} = 1+\gamma$</td>
<td>$k_{a1} = 1/(1-\gamma)$, $k_{d1} = -\gamma/(1-\gamma)$</td>
</tr>
<tr>
<td>$\gamma_{11} = -\gamma/(1-\gamma^2)$, $\gamma_{12} = 1$, $\gamma_{21} = 1$, $\gamma_{22} = \gamma$</td>
<td>$\gamma_{11} = 1$, $\gamma_{12} = 1$, $\gamma_{21} = -(1-\gamma^2)/\gamma$, $\gamma_{22} = \gamma$</td>
<td>$\gamma_{11} = 1$, $\gamma_{12} = -(1-\gamma^2)/\gamma$, $\gamma_{21} = 1$, $\gamma_{22} = \gamma$</td>
</tr>
</tbody>
</table>

Fig. 7. Diagram of the N-order branch of the modified lattice wave digital filter realized by first-order all-pass sections
As an example, the realization an \( N \)th order branch of the lattice filter with modified first-order all-pass sections is depicted in Fig. 7. The resulting value of the overall branch coefficient can be calculated as
\[
\gamma_s = \prod_{i=1}^{N} \frac{k_{di}}{k_{wi}}.
\]

In Figure 8 the block diagram of the modified lattice wave digital filter is depicted.

Authors’ realization of the modified first-order section with the TMS320C40 digital signal processor is presented in Fig. 9.

As an illustrative example a bireciprocal lattice modified wave digital filter was designed. The filter parameters (chosen by authors) are: stopband normalized frequency \( F_z = 0.29 \), stopband attenuation \( \delta_z = 55 \text{ dB} \). The passband parameters result from the bireciprocal conditions: passband attenuation equals \( \delta_p = 13.7 \times 10^{-6} \text{ dB} \) and the passband normalized frequency is \( F_p = 0.21 \). The filter coefficients are calculated using Matlab functions written by authors [6].

Fig. 10. Block diagram of the designed bireciprocal modified lattice wave digital filter realized with TMS320C40
The calculated values of the coefficients are: \( \gamma_0 = 0, \gamma_1 = -0.0919947, \gamma_2 = 0, \gamma_3 = -0.317108, \gamma_4 = 0, \gamma_5 = -0.584461, \gamma_6 = 0, \gamma_7 = -0.854180, \gamma_8 = 0 \). The resulting values of the branch coefficients are

\[
\gamma_{s1} = \left(1 - \gamma_1^2\right)\left(1 - \gamma_2^2\right) = 0.6528334, \quad \gamma_{s2} = \left(1 - \gamma_3^2\right)\left(1 - \gamma_4^2\right) = 0.243188.
\]

(6ab)

Fig. 11. Frequency response of the designed bireciprocal modified lattice wave digital filter realized with TMS320C40: (a) magnitude response, (b) phase response

3. Interpolator

A special class of lattice wave digital filters referred to as bireciprocal, is suitable for the realization of interpolators. Characteristic function \( K(\psi) \) of bireciprocal filters satisfies equation

\[
K(\psi) = \frac{1}{K\left(\frac{1}{\psi}\right)}, \quad \text{where} \quad \psi = \frac{z-1}{z+1}.
\]

(7)

For this kind of filters every even filter coefficient is equal to zero and the filter circuit is simplified. Block diagrams of two versions of the bireciprocal lattice wave digital filters are shown in Fig. 12. The adaptors are realized as shown in Fig. 3. In the first version (Fig. 12a) for summing the output signals of the branches \( S_1 \) and \( S_2 \) an adder is used. By replacing the output adder (Fig. 12b) by a switch it is possible to double the output signal sampling rate. Bireciprocal lattice wave digital filters of this kind are very useful for building interpolators. A cascaded version of such an interpolator with bireciprocal modified lattice wave digital filters is shown in Fig. 13. The filter branches are realized as in Fig. 9.

As an illustrative example a cascaded interpolator for a class-D power audio amplifier [3, 6] is used. Parameters chosen by the authors for this interpolator are: passband ripple \( \delta_p < 0.1 \) dB, oversampling ratio \( R = 8 \), passband 4...20 kHz, signal-to-noise and distortion ratio \( S_{\text{INAD}} < 90 \) dB. Design parameters for the interpolator stages are described in Table 2. Authors applied bireciprocal lattice wave digital elliptic filters for this realization. Filter coefficients are designed with authors’ program prepared in the Matlab environment, based on the methods presented in [6].
Fig. 12. Block diagrams of two versions of bireciprocal lattice wave digital filters: (a) with adder at the output, (b) with switch at the output

Fig. 13. Cascaded version of the interpolator for $R = 8$

Fig. 14. Block diagram of the cascaded version of the interpolator with a single switch and the resulting multipliers
Table 2. Design parameters for interpolator stages

<table>
<thead>
<tr>
<th>Stage</th>
<th>Order</th>
<th>( F_p )</th>
<th>( F_z )</th>
<th>( \delta_p ) [dB]</th>
<th>( \delta_z ) [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>0.2267</td>
<td>0.2733</td>
<td>3.2 ( \cdot ) 10^{-9}</td>
<td>90</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>0.1134</td>
<td>0.3866</td>
<td>1.1 ( \cdot ) 10^{-9}</td>
<td>90</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.0567</td>
<td>0.4433</td>
<td>6 ( \cdot ) 10^{-10}</td>
<td>90</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>0.0567</td>
<td>0.0683</td>
<td>7.5 ( \cdot ) 10^{-9}</td>
<td>90</td>
</tr>
</tbody>
</table>

The interpolator was realized with TMS320C40 signal processor by Texas Instruments using modified wave digital filters. The structure of the interpolator is depicted in Fig. 14. The resulting value of the coefficient \( \gamma_{sw1} \) is given by the following equation

\[
\gamma_{sw1} = \gamma_{s12} \gamma_{s22} \gamma_{s32},
\]

where \( \gamma_{s12}, \gamma_{s22}, \gamma_{s32} \) are the resultant coefficients of upper branches for stages 1, 2, 3, respectively. Other resultant coefficients \( \gamma_{sw2} \ldots \gamma_{sw8} \) can be similarly calculated. Frequency response of the cascaded interpolator realized with the TMS320C40 for \( R = 8 \) is shown in Fig. 15. The interpolator achieves the signal-to-noise and distortion ratio \( S_{INAD} \) near to 90dB and the passband ripple of \( \delta_p \approx 8 \cdot 10^{-9} \)dB.

![Frequency response](image)

Fig. 15. Frequency response of the cascaded interpolator realized by TMS320C40 for \( R = 8 \): (a), (c), (d) magnitude response, (b) phase response

4. Conclusions

The presented modified wave digital filters are very efficient for the implementation in modern floating-point digital signal processors such as the TMS320C40 processor and the VLIW architecture TMS320C6000 processor. They are especially important for large dynamic range applications. Similarly modified ladder wave digital filters can be designed.

References

