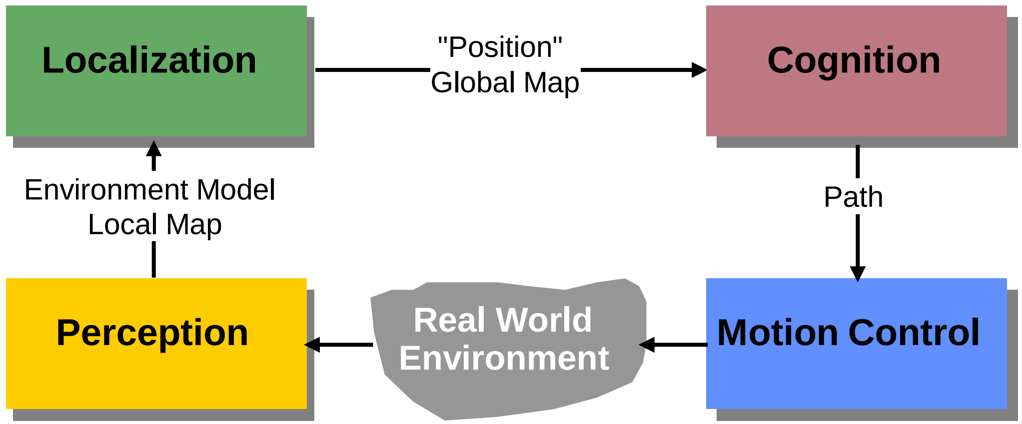


Mobile Robot Kinematics

Mobile Robot Kinematics: Overview

- Mobile robot and manipulator arm characteristics
  - Arm is fixed to the ground and usually comprised of a single chain of actuated links
  - Mobile robot motion is defined through rolling and sliding constra-ints taking effect at the wheel-ground contact points

C Willow Garage

C dexter1232222222222, youtube.com

3 Mobile Robot Kinematics: Overview

- Definition and Origin
  - From *kinein* (Greek); to move
  - Kinematics is the subfield of Mechanics which deals with motions of bodies

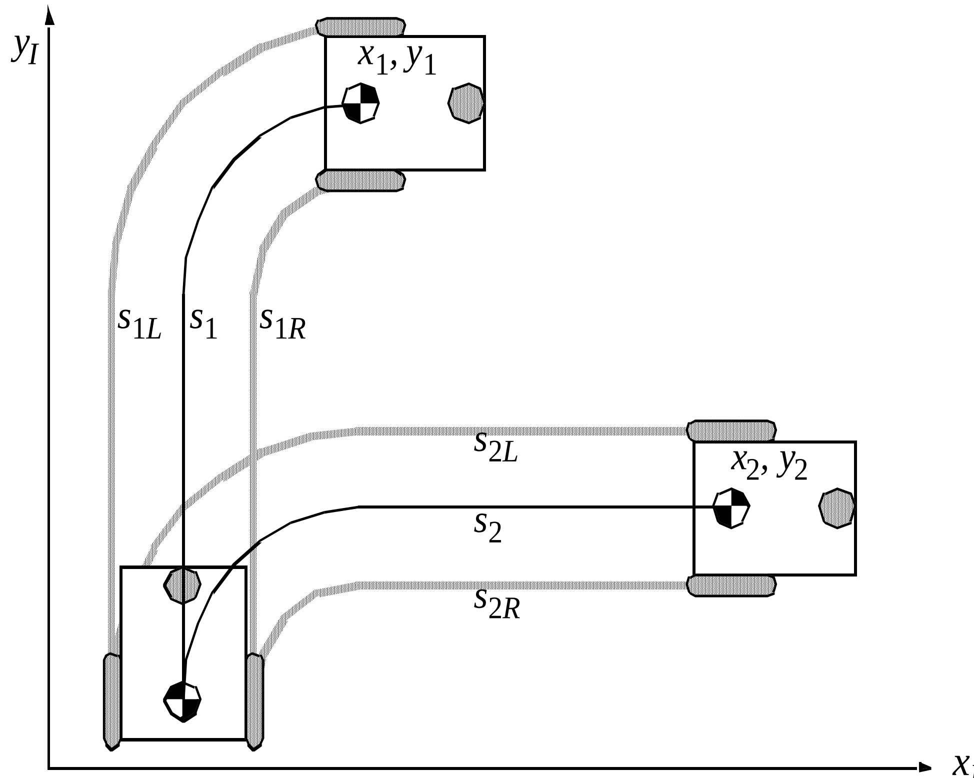
- Manipulator- vs. Mobile Robot Kinematics
  - Both are concerned **with forward and inverse kinematics**
  - However, for mobile robots, encoder values don‘t map to unique robot poses
  - However, **mobile robots** can move unbound with respect to their environment
    - There is **no direct** (=instantaneous) **way to measure the robot’s position**
    - **Position must be integrated over time**, depends on path taken
    - Leads to inaccuracies of the position (motion) estimate
  - Understanding mobile robot motion starts with **understanding wheel con-straints** placed on the robot’s mobility

Non-Holonomic Systems

- Non-holonomic systems
  - differential equations are not integrable to the final position.
  - the measure of the traveled distance of each wheel is not sufficient to calculate the final position of the robot. One has also to know how this movement was executed as a function of time.
  - This is in stark contrast to actuator arms

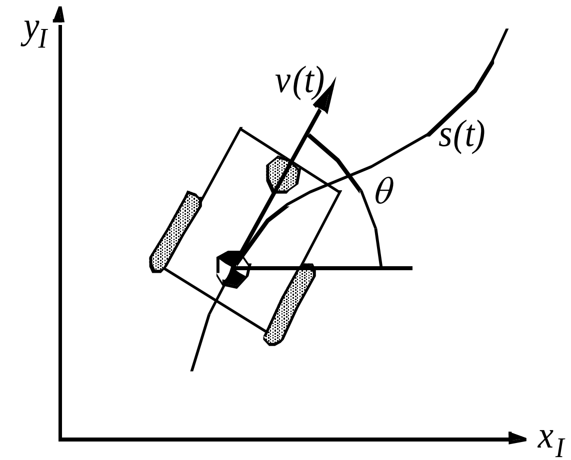
$$s_1 = s_2, s_{1R} = s_{2R}, s_{1L} = s_{2L}$$

$$x_1 \neq x_2, y_1 \neq y_2$$



Non-Holonomic Systems

- A mobile robot is running along a trajectory *s(t)*.
  At every instant of the movement its velocity *v(t)* is:



$$v(t) = \frac{\partial s}{\partial t} = \frac{\partial x}{\partial t}\cos\theta + \frac{\partial y}{\partial t}\sin\theta$$

$$ds = dx\cos\theta + dy\sin\theta$$

- Function *v(t)* is said to be integrable (holonomic) if there exists a trajectory function *s(t)* that can be described by the values *x*, *y*, and θ only.

$$s = s(x, y, \theta)$$

- This is the case if

$$\frac{\partial^2 s}{\partial x \partial y} = \frac{\partial^2 s}{\partial y \partial x} \quad ; \quad \frac{\partial^2 s}{\partial x \partial \theta} = \frac{\partial^2 s}{\partial \theta \partial x} \quad ; \quad \frac{\partial^2 s}{\partial y \partial \theta} = \frac{\partial^2 s}{\partial \theta \partial y}$$

Condition for s to be integrable function

Non-Holonomic Systems

- With $s = s(x,y,\theta)$ we get for ds

$$ds = \frac{\partial s}{\partial x}dx + \frac{\partial s}{\partial y}dy + \frac{\partial s}{\partial \theta}d\theta$$

- and by comparing the equation above with $ds = dx\cos\theta + dy\sin\theta$

- we find $\frac{\partial s}{\partial x} = \cos\theta \quad ; \quad \frac{\partial s}{\partial y} = \sin\theta \quad ; \quad \frac{\partial s}{\partial \theta} = 0$

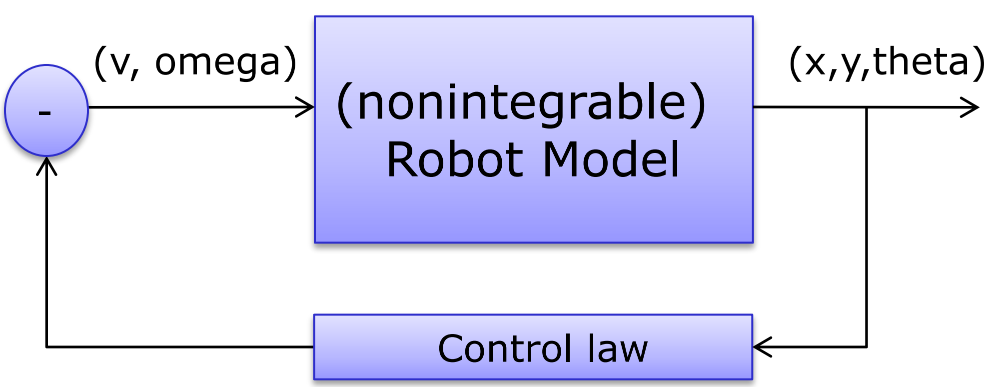
- Condition for an integrable (holonomic) function:
  - the second (-sinθ=0) and third (cosθ=0) term in the equation do not hold!

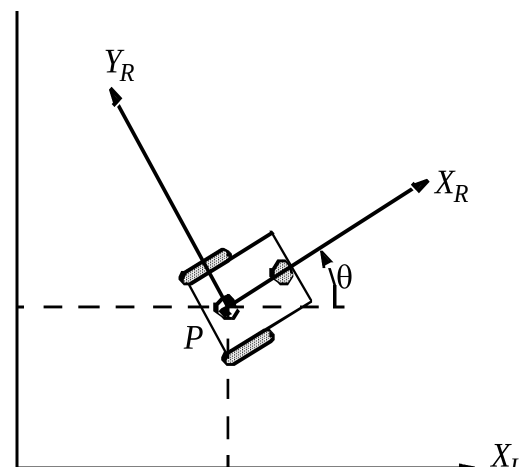
$$\frac{\partial^2 s}{\partial x \partial y} = \frac{\partial^2 s}{\partial y \partial x} \quad ; \quad \frac{\partial^2 s}{\partial x \partial \theta} = \frac{\partial^2 s}{\partial \theta \partial x} \quad ; \quad \frac{\partial^2 s}{\partial y \partial \theta} = \frac{\partial^2 s}{\partial \theta \partial y}$$

Forward and Inverse Kinematics

- Forward kinematics:
  - Transformation from joint- to physical space
- Inverse kinematics
  - Transformation from physical- to joint space
  - Required for motion control

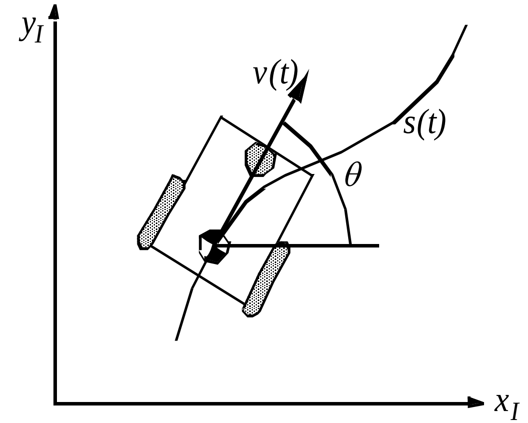
- Due to nonholonomic constraints in mobile robotics, we deal with **differential** (inverse) kinematics
  - Transformation between velocities instead of positions
  - Such a differential kinematic model of a robot has the following form:





Differential Kinematics Model

- Due to a lack of alternatives:
  - establish the robot speed $\dot{\xi} = \begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T$ as a function of the wheel speeds $\dot{\varphi}_i$, steering angles β_i , steering speeds $\dot{\beta}_i$ and the geometric parameters of the robot (*configuration coordinates*).
  - forward kinematics



$$\dot{\xi} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(\dot{\varphi}_1, \ldots \dot{\varphi}_n, \beta_1, \ldots \beta_m, \dot{\beta}_1, \ldots \dot{\beta}_m)$$

  - Inverse kinematics

$$\begin{bmatrix} \dot{\varphi}_1 & \cdots & \dot{\varphi}_n & \beta_1 & \ldots & \beta_m & \dot{\beta}_1 & \ldots & \dot{\beta}_m \end{bmatrix}^T = f(\dot{x}, \dot{y}, \dot{\theta})$$

  - But generally not integrable into

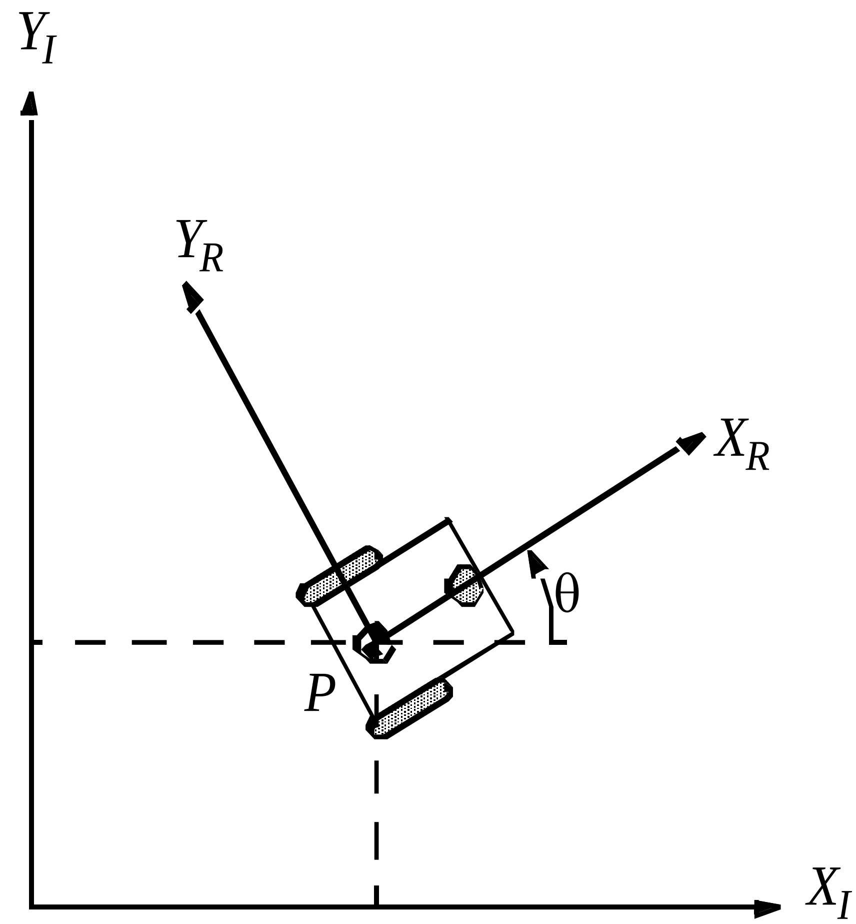
$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = f(\varphi_1, \ldots \varphi_n, \beta_1, \ldots \beta_m)$$

Representing Robot Pose

- Representing the robot within an arbitrary initial frame
  - Inertial frame: $\{X_I, Y_I\}$
  - Robot frame: $\{X_R, Y_R\}$
  - Robot pose: $\xi_I = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$
  - Mapping between the two frames

$$\dot{\xi}_R = R(\theta)\dot{\xi}_I = R(\theta) \cdot \begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T$$

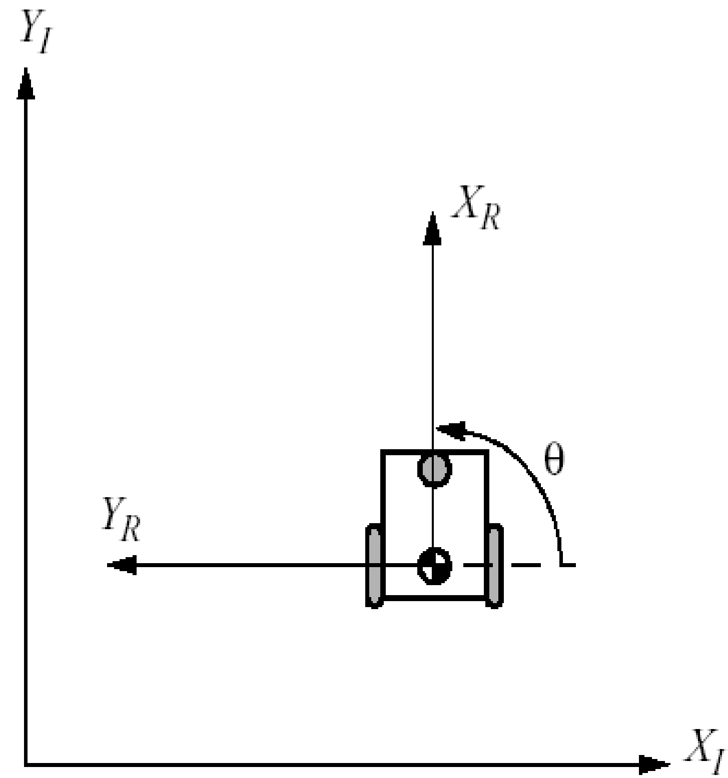
$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Example: Robot aligned with Y_I

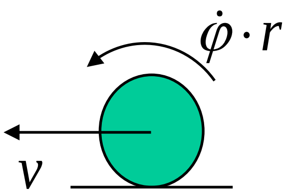
$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

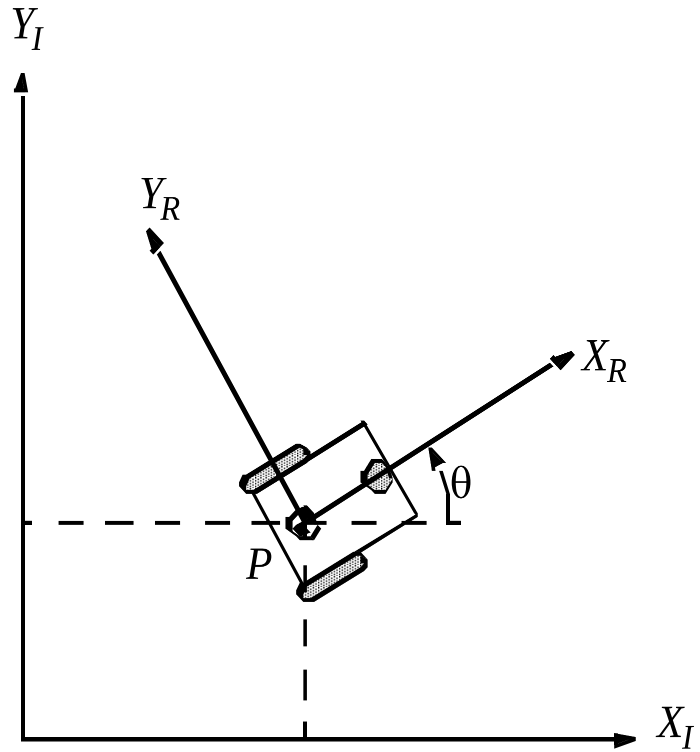
$$\dot{\xi}_R = R(\frac{\pi}{2})\dot{\xi}_I = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\dot{x} \\ \dot{\theta} \end{bmatrix}$$



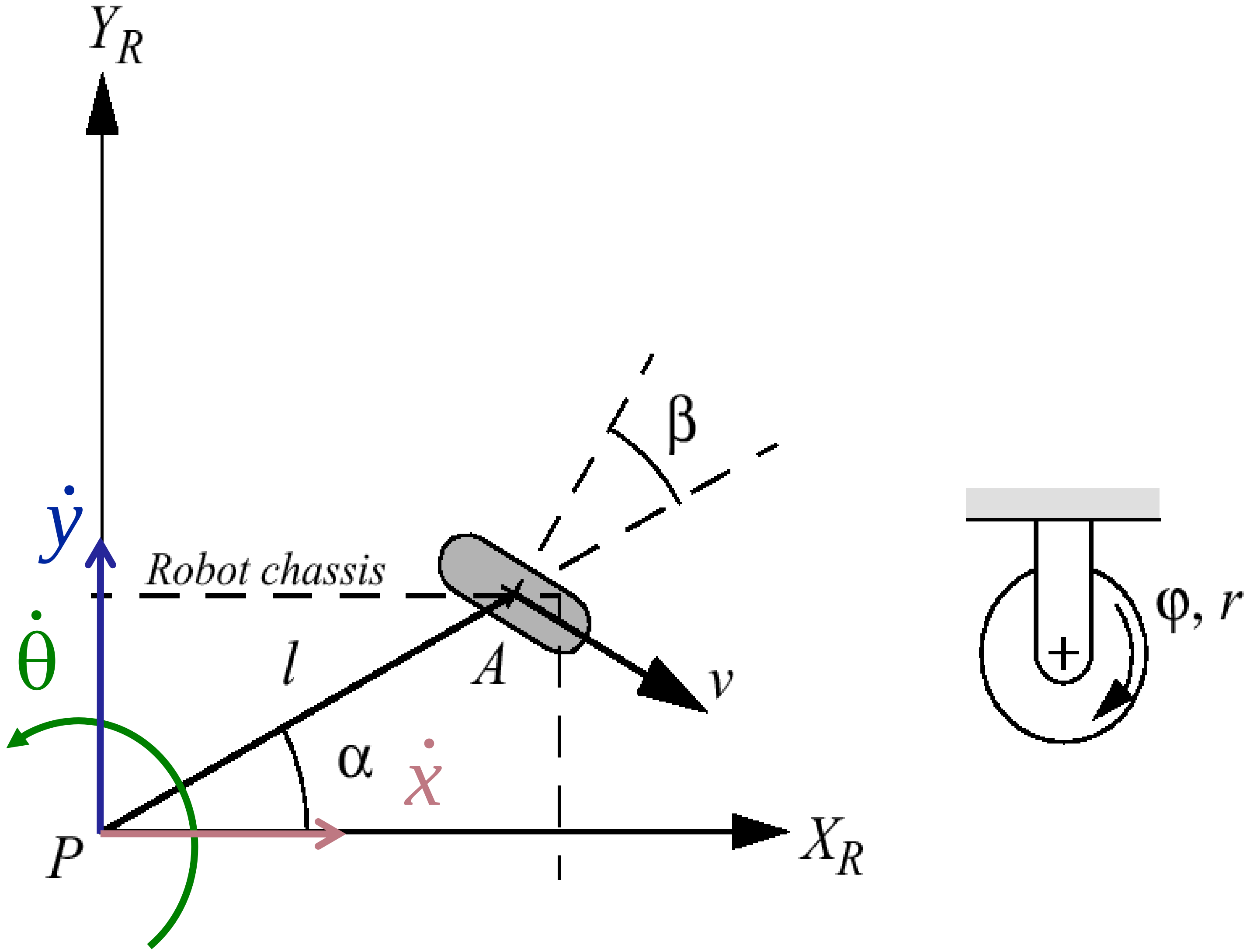
Wheel Kinematic Constraints

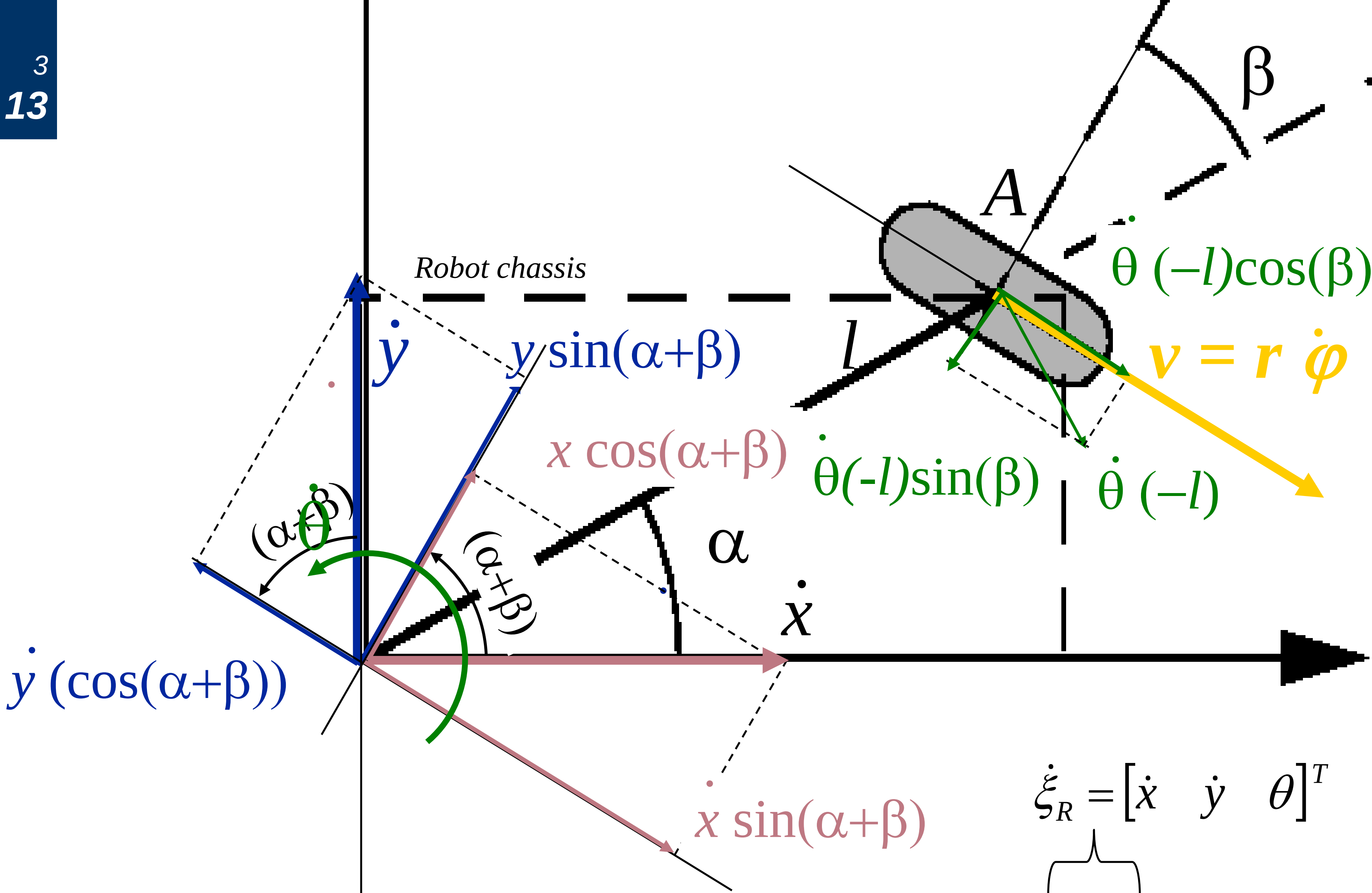
- Assumptions
  - Movement on a horizontal plane
  - Point contact of the wheels
  - Wheels not deformable
  - Pure rolling (v_c = 0 at contact point)
  - No slipping, skidding or sliding
  - No friction for rotation around contact point
  - Steering axes orthogonal to the surface
  - Wheels connected by rigid frame (chassis)





Kinematic Constraints: Fixed Standard Wheel





$$\dot{\xi}_R = \begin{bmatrix} \dot{x} & \dot{y} & \theta \end{bmatrix}^T$$

$$\begin{bmatrix} \sin(\alpha+\beta) & -\cos(\alpha+\beta) & (-l)\cos\beta \end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

$$\begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) & l\sin\beta \end{bmatrix} R(\theta)\dot{\xi}_I = 0$$

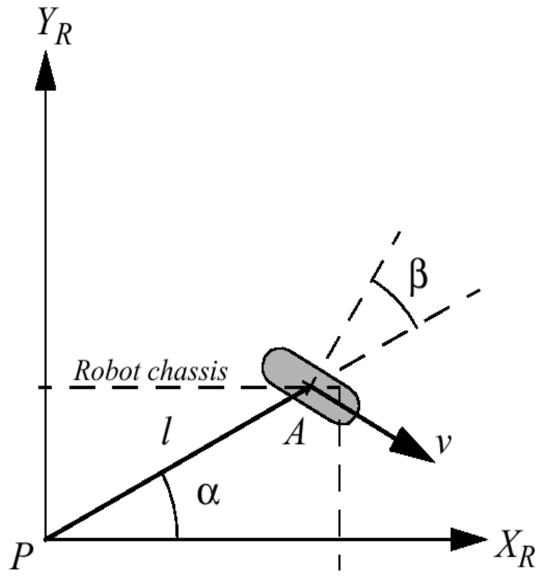
Example

$$\left[\sin(\alpha+\beta)\ \ -\cos(\alpha+\beta)\ \ (-l)\cos\beta\right] R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

$$\left[\cos(\alpha+\beta)\ \ \sin(\alpha+\beta)\ \ l\sin\beta\right] R(\theta)\dot{\xi}_I = 0$$

- Suppose that the wheel A is in position such that **α = 0 and β = 0**
- This would place the contact point of the wheel on X_I with the plane of the wheel oriented parallel to Y_I. If θ = 0, then the **sliding constraint** reduces to:

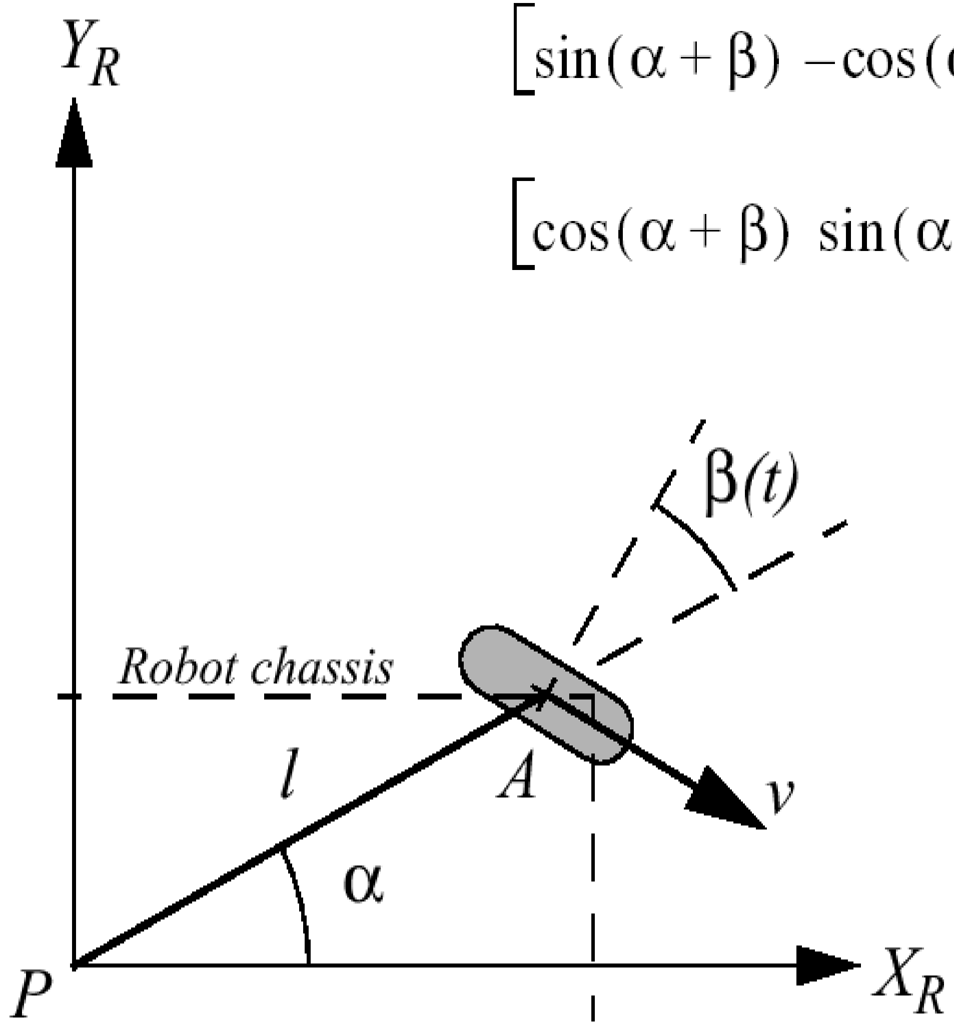
$$\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = 0$$

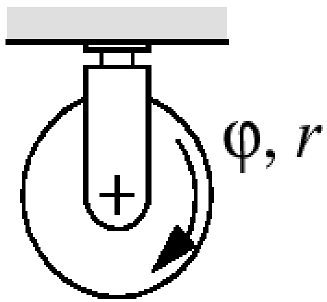


Kinematic Constraints: Steered Standard Wheel

$$\begin{bmatrix} \sin(\alpha+\beta) & -\cos(\alpha+\beta) & (-l)\cos\beta \end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

$$\begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) & l\sin\beta \end{bmatrix} R(\theta)\dot{\xi}_I = 0$$

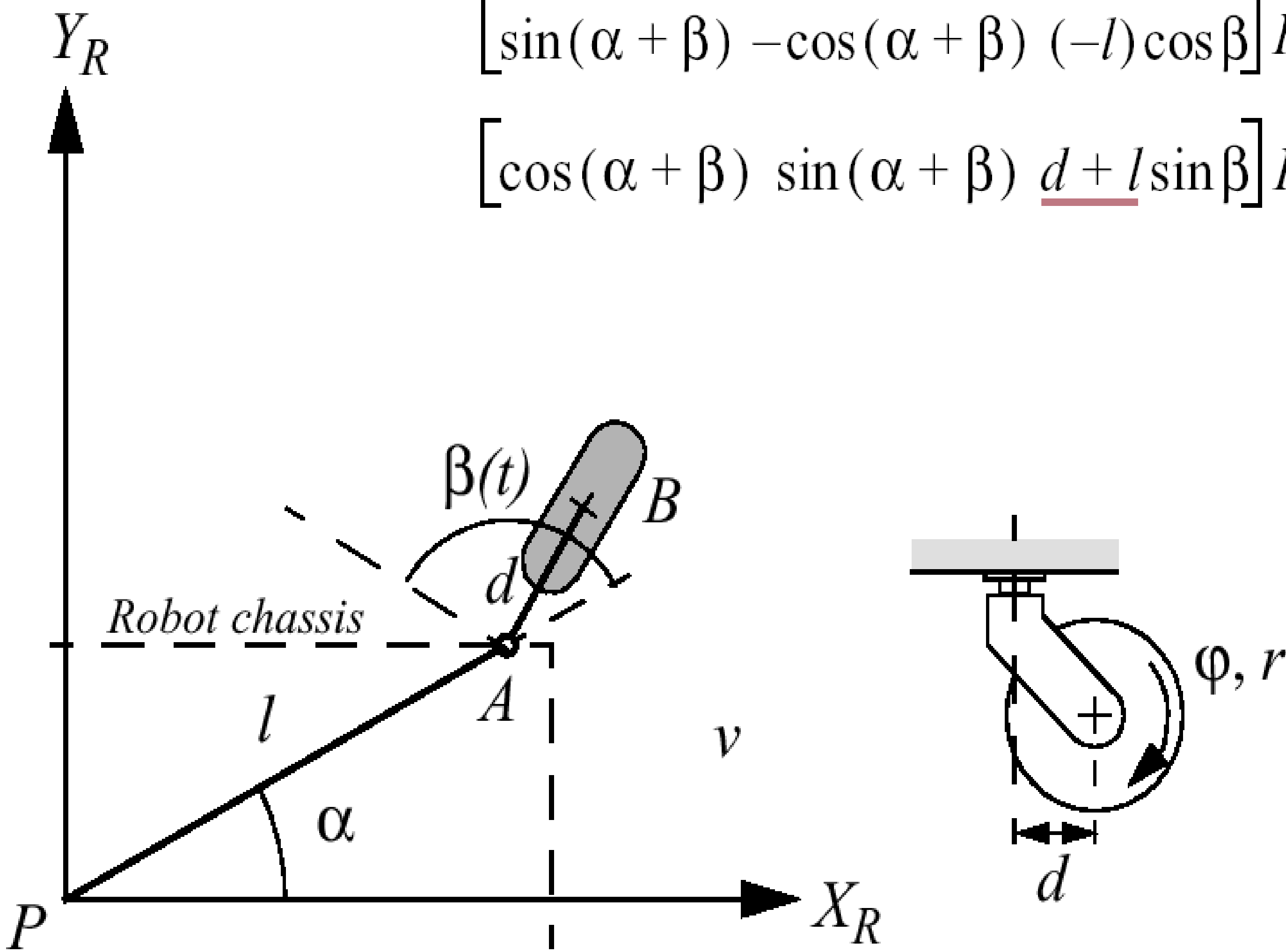




Kinematic Constraints: Castor Wheel

$$\left[\sin(\alpha+\beta) \;\; -\cos(\alpha+\beta) \;\; (-l)\cos\beta\right] R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

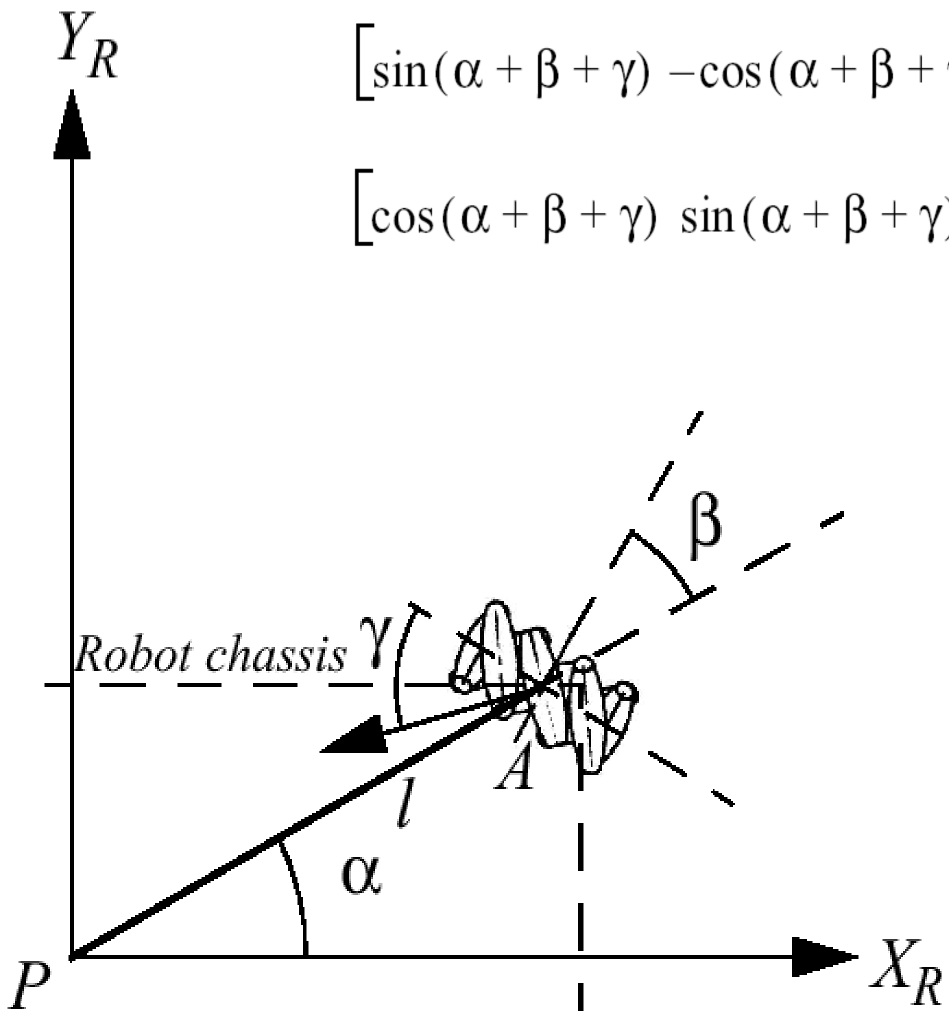
$$\left[\cos(\alpha+\beta) \;\; \sin(\alpha+\beta) \;\; d + l\sin\beta\right] R(\theta)\dot{\xi}_I + d\dot{\beta} = 0$$

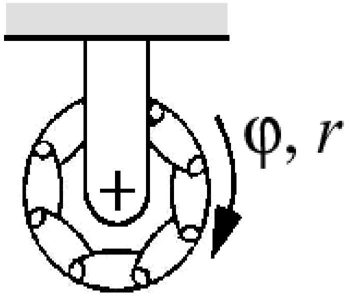


Kinematic Constraints: Swedish Wheel

$$\begin{bmatrix}\sin(\alpha+\beta+\gamma) & -\cos(\alpha+\beta+\gamma) & (-l)\cos(\beta+\gamma)\end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi}\cos\gamma = 0$$

$$\begin{bmatrix}\cos(\alpha+\beta+\gamma) & \sin(\alpha+\beta+\gamma) & l\sin(\beta+\gamma)\end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi}\sin\gamma - r_{sw}\dot{\varphi}_{sw} = 0$$

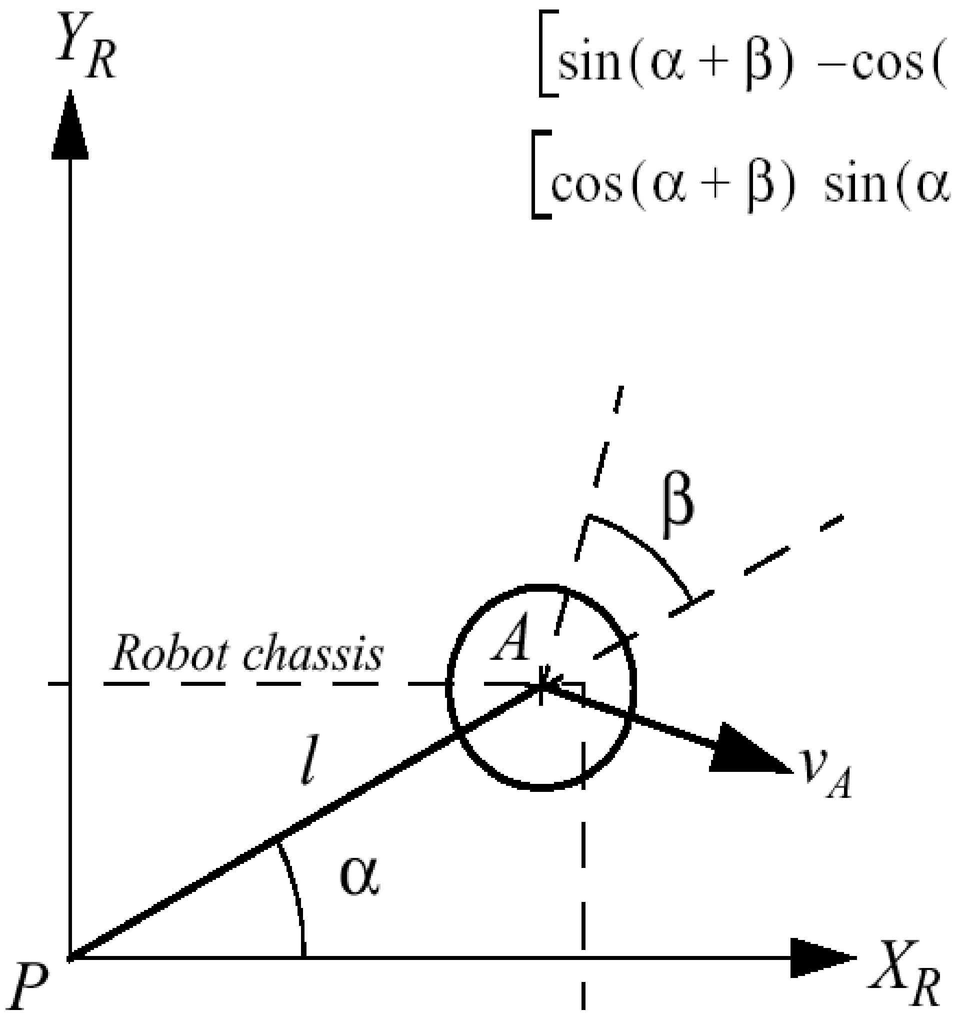


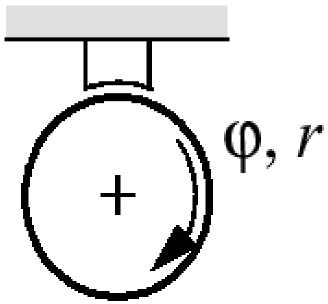


Kinematic Constraints: Spherical Wheel

$$\begin{bmatrix} \sin(\alpha+\beta) & -\cos(\alpha+\beta) & (-l)\cos\beta \end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

$$\begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) & l\sin\beta \end{bmatrix} R(\theta)\dot{\xi}_I = 0$$





- Rotational axis of the wheel can have an arbitrary direction

Kinematic Constraints: Complete Robot

- Given a robot with *M* wheels
  - each wheel imposes zero or more constraints on the robot motion
  - **only fixed and steerable standard wheels impose constraints**
- What is the maneuverability of a robot considering a combination of different wheels?
- Suppose we have a total of $N = N_f + N_s$ standard wheels
  - We can develop the equations for the constraints in matrix forms:

  - Rolling

$$J_1(\beta_s)R(\theta)\dot{\xi}_I + J_2\dot{\varphi} = 0 \qquad \varphi(t) = \begin{bmatrix} \varphi_f(t) \\ \varphi_s(t) \end{bmatrix}_{(N_f+N_s)\times 1} \qquad J_1(\beta_s) = \begin{bmatrix} J_{1f} \\ J_{1s}(\beta_s) \end{bmatrix}_{(N_f+N_s)\times 3} \qquad J_2 = diag(r_1 \cdots r_N)$$

  - Lateral movement

$$C_1(\beta_s)R(\theta)\dot{\xi}_I = 0 \qquad C_1(\beta_s) = \begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix}_{(N_f+N_s)\times 3}$$

Mobile Robot Maneuverability

- The maneuverability of a mobile robot is the combination
  - of the mobility available based on the sliding constraints
  - plus additional freedom contributed by the steering

- Three wheels is sufficient for static stability
  - additional wheels need to be synchronized
  - this is also the case for some arrangements with three wheels

- It can be derived using the equation seen before
  - Degree of mobility δ_m
  - Degree of steerability δ_s
  - Robots maneuverability $\delta_M = \delta_m + \delta_s$

Mobile Robot Maneuverability: Degree of Mobility

- To avoid any lateral slip the motion vector $R(\theta)\dot{\xi}_I$ has to satisfy the following constraints:

$$C_{1f} R(\theta)\dot{\xi}_I = 0$$

$$C_{1s}(\beta_s) R(\theta)\dot{\xi}_I = 0$$

$$C_1(\beta_s) = \begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix}$$

- Mathematically:
  - $R(\theta)\dot{\xi}_I$ must belong to the *null space* of the projection matrix $C_1(\beta_s)$
  - *Null space* of $C_1(\beta_s)$ is the space N such that for any vector n in N

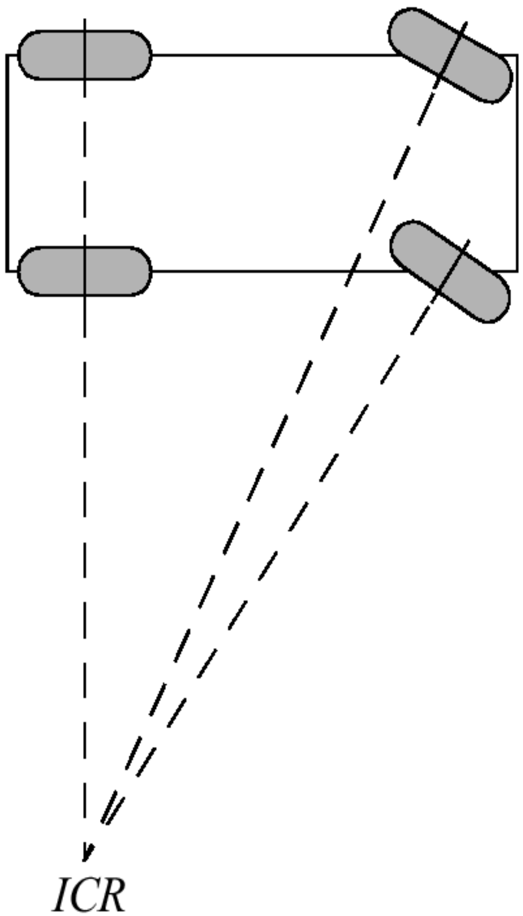
$$C_1(\beta_s) \cdot n = 0$$

  - Geometrically this can be shown by the *Instantaneous Center of Rotation (ICR)*

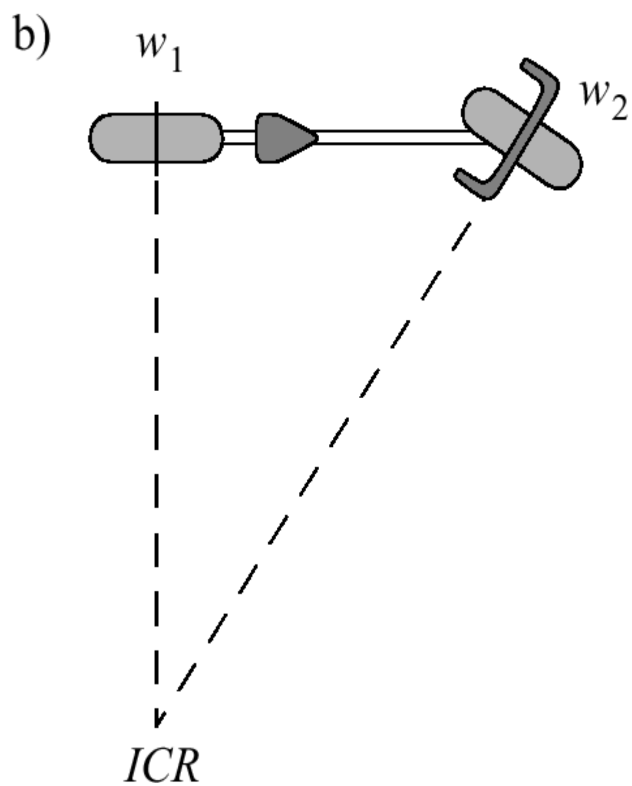
Mobile Robot Maneuverability: ICR

- Instantaneous center of rotation (ICR)

- Ackermann Steering



Bicycle



Mobile Robot Maneuverability: More on Degree of Mobility

- Robot chassis kinematics is a function of the set of independent constraints

$$rank\left[C_1(\beta_s)\right] \qquad C_1(\beta_s)=\begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix} \qquad \begin{matrix} C_{1f}R(\theta)\dot{\xi}_I=0 \\ C_{1s}(\beta_s)R(\theta)\dot{\xi}_I=0 \end{matrix}$$

  - the greater the rank of $C_1(\beta_s)$ the more constrained is the mobility

- Mathematically

$$\delta_m=\dim N\left[C_1(\beta_s)\right]=3-rank\left[C_1(\beta_s)\right] \qquad 0\le rank\left[C_1(\beta_s)\right]\le 3$$

  - no standard wheels $\quad rank\left[C_1(\beta_s)\right]=0$
  - all direction constrained $\quad rank\left[C_1(\beta_s)\right]=3$

- Examples:
  - Unicycle: One single fixed standard wheel
  - Differential drive: Two fixed standard wheels
    - wheels on same axle
    - wheels on different axle

24 Mobile Robot Maneuverability: Degree of Steerability

- Indirect degree of motion

$$\delta_s = rank\left[C_{1s}(\beta_s)\right]$$

  - The particular orientation at any instant imposes a kinematic constraint
  - However, the ability to change that orientation can lead additional degree of maneuverability
- Range of δ_s: $0 \le \delta_s \le 2$

- Examples:
  - one steered wheel: Tricycle
  - two steered wheels: No fixed standard wheel
  - car (Ackermann steering): N_f = 2, N_s=2 -> common axle

Mobile Robot Maneuverability: Robot Maneuverability

- Degree of Maneuverability

$$\delta_M = \delta_m + \delta_s$$

  - Two robots with same δ_M are not necessary equal
  - Example: Differential drive and Tricycle (next slide)

  - For any robot with $\delta_M = 2$ the ICR is always constrained to *lie on a line*
  - For any robot with $\delta_M = 3$ the ICR is not constrained and can *be set to any point on the plane*

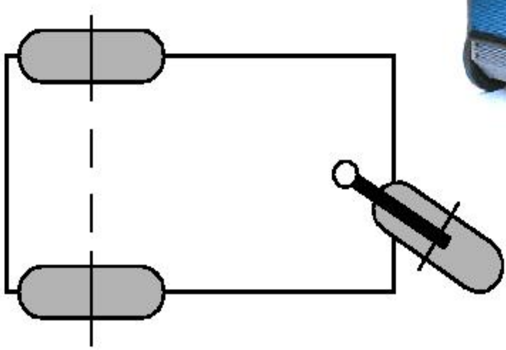
- The Synchro Drive example: $\delta_M = \delta_m + \delta_s = 1 + 1 = 2$

Mobile Robot Maneuverability: Wheel Configurations

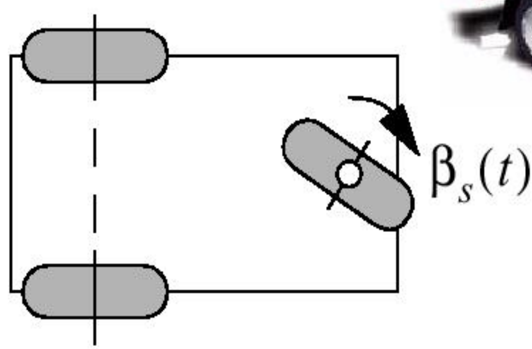
- Differential Drive

Tricycle

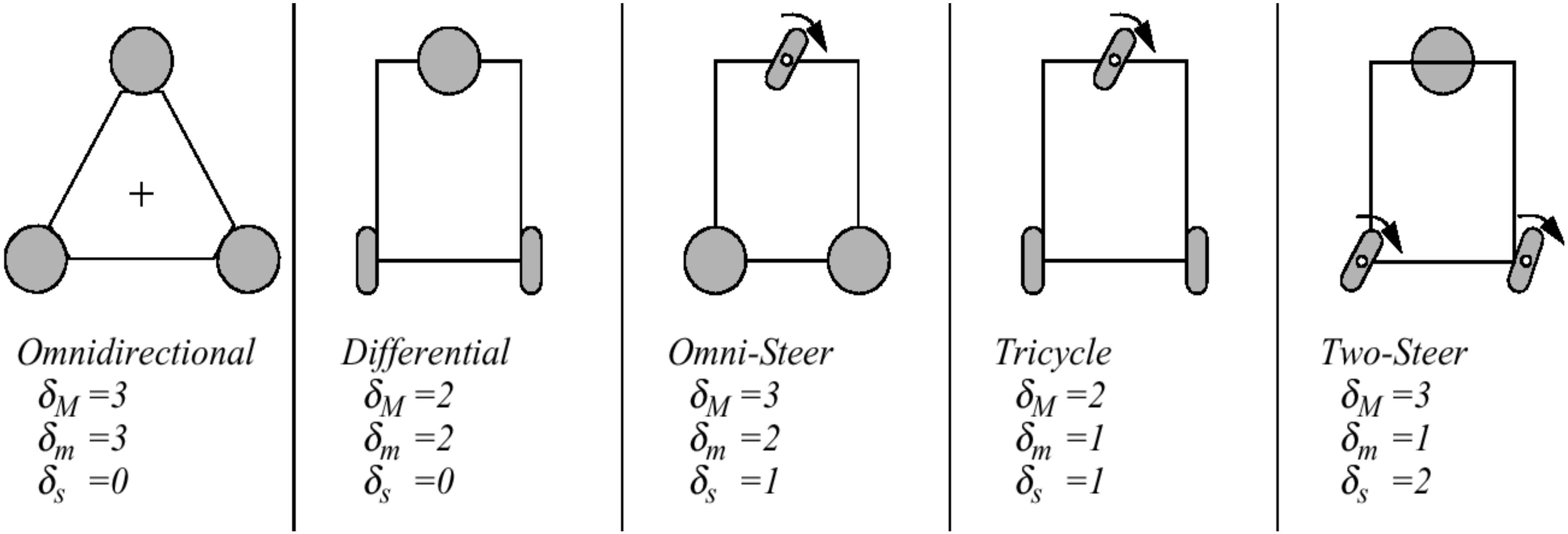
a)



b)

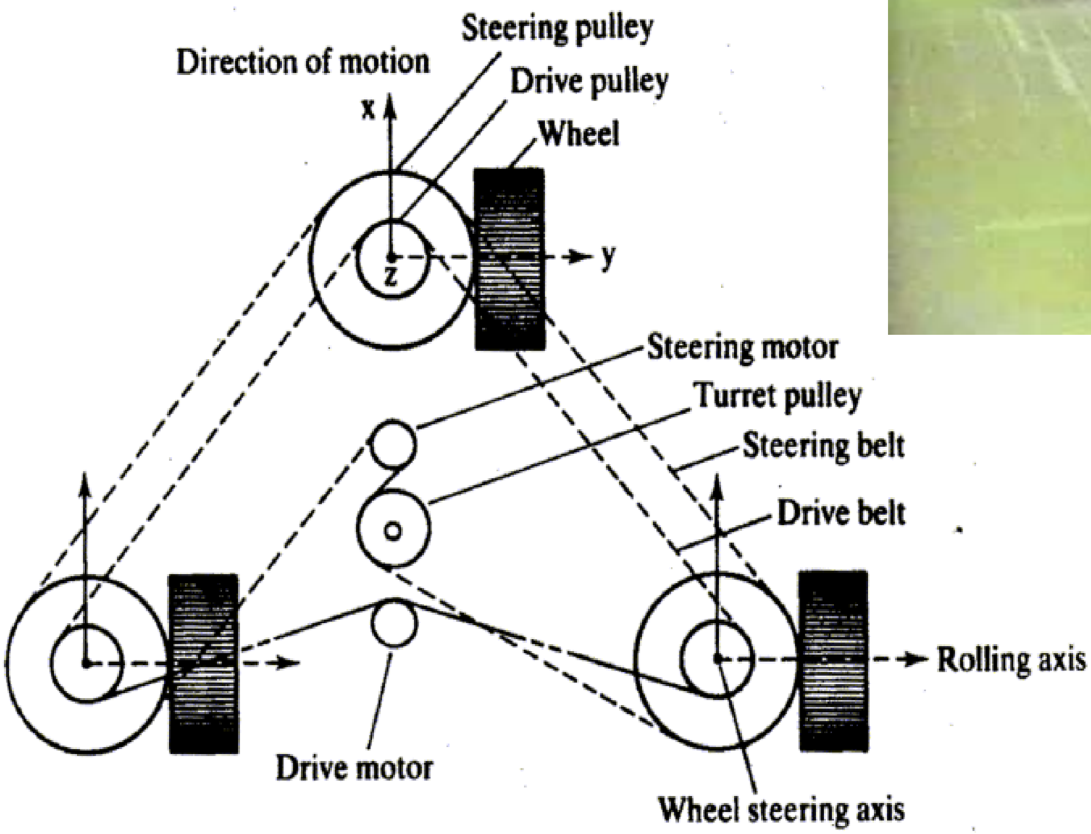


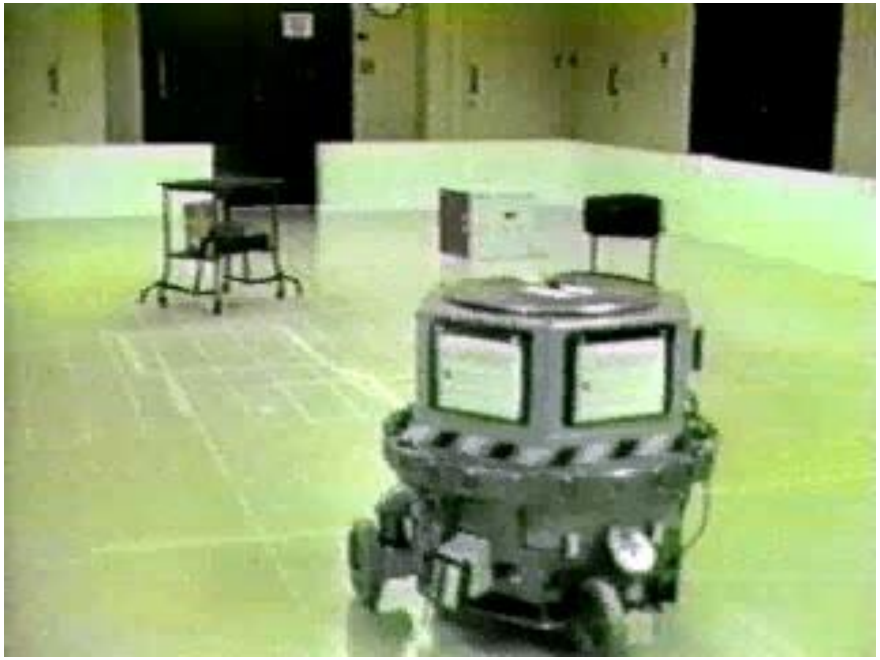
Five Basic Types of Three-Wheel Configurations



Synchro Drive

$$\delta_M = \delta_m + \delta_s = 1 + 1 = 2$$





C J. Borenstein

Mobile Robot Workspace: Degrees of Freedom

- The Degree of Freedom (DOF) is the robot's ability to achieve various poses.

- But what is the degree of vehicle's freedom in its environment?
  - Car example

- Workspace
  - how the vehicle is able to move between different configuration in its workspace?

- The robot's independently achievable velocities
  - = *differentiable degrees of freedom (DDOF) =* δ_m
  - Bicycle: $\delta_M = \delta_m + \delta_s = 1 + 1$ DDOF = 1; DOF=3
  - Omni Drive: $\delta_M = \delta_m + \delta_s = 3 + 0$ DDOF=3; DOF=3

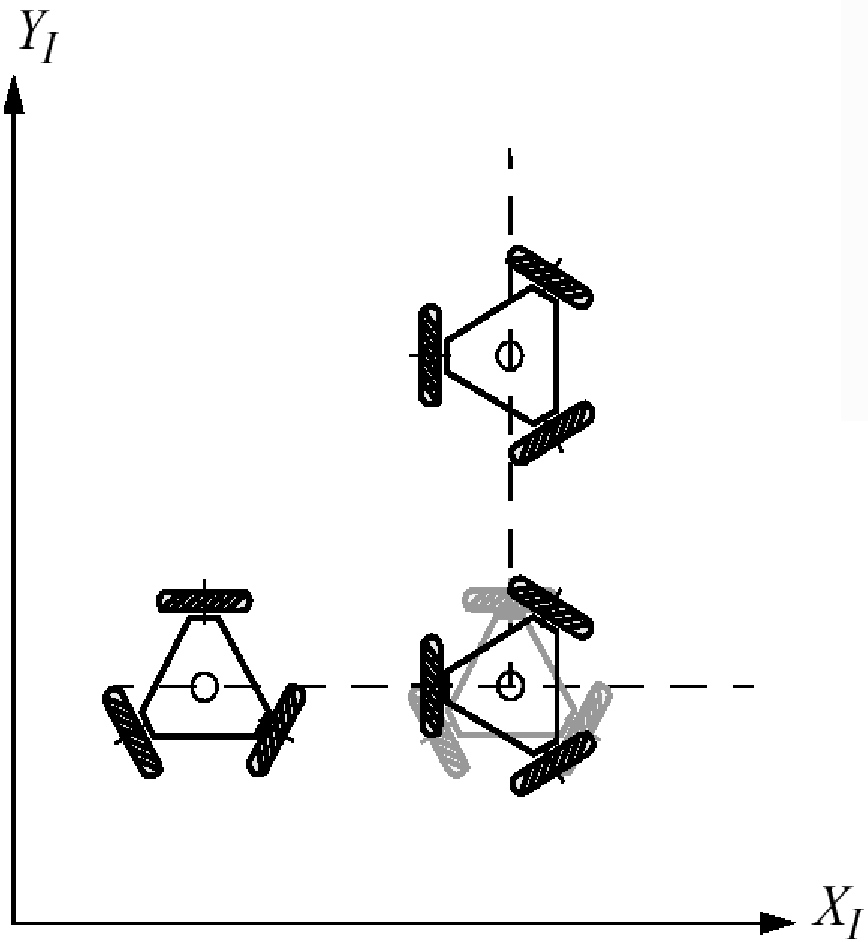
Mobile Robot Workspace: Degrees of Freedom, Holonomy

- DOF *degrees of freedom*:
  - Robots ability to achieve various poses
- DDOF *differentiable degrees of freedom*:
  - Robots ability to achieve various trajectories

$$DDOF \leq \delta_M \leq DOF$$

- Holonomic Robots
  - A holonomic kinematic constraint can be expressed as an explicit function of position variables only
  - A non-holonomic constraint requires a different relationship, such as the derivative of a position variable
  - *Fixed and steered standard wheels impose non-holonomic constraints*

Path / Trajectory Considerations: Omnidirectional Drive



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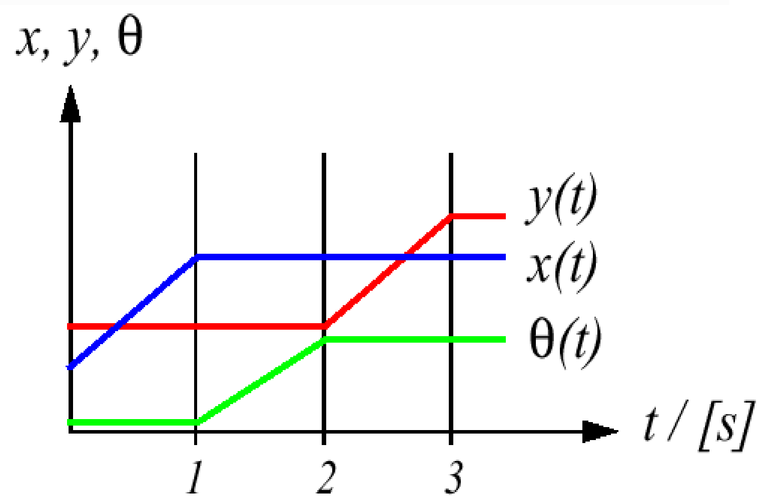
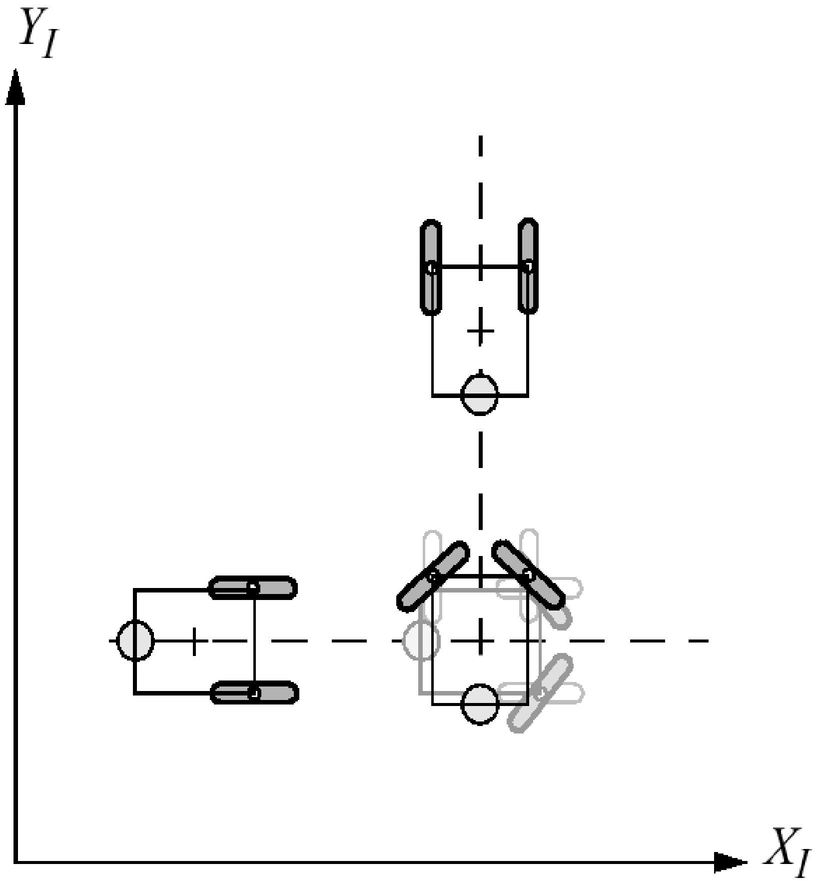
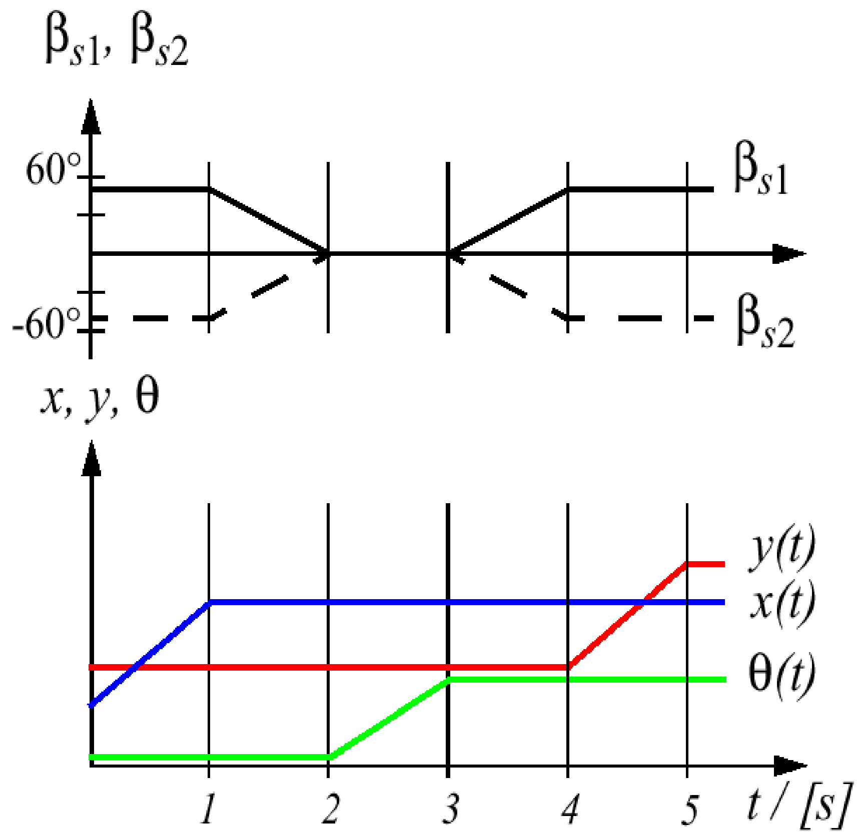


Path / Trajectory Considerations: Two-Steer





Beyond Basic Kinematics

- At higher speeds, and in difficult terrain, dynamics become important

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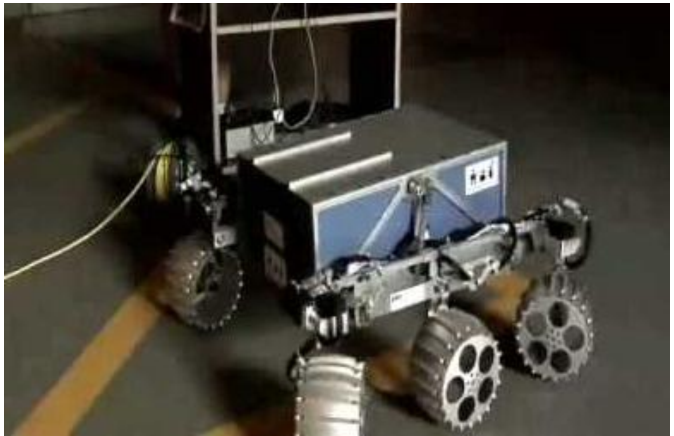
- For other vehicles, the no-sliding constraints, and simple kinematics presented in this lecture do not hold



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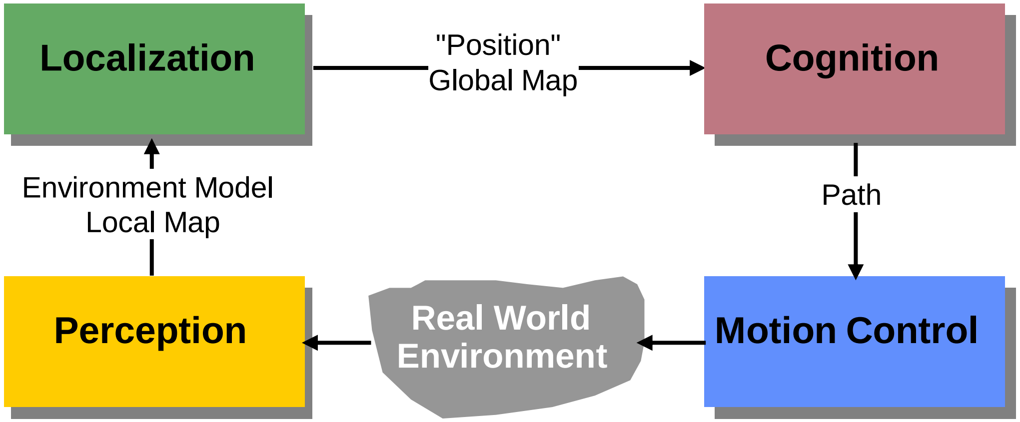


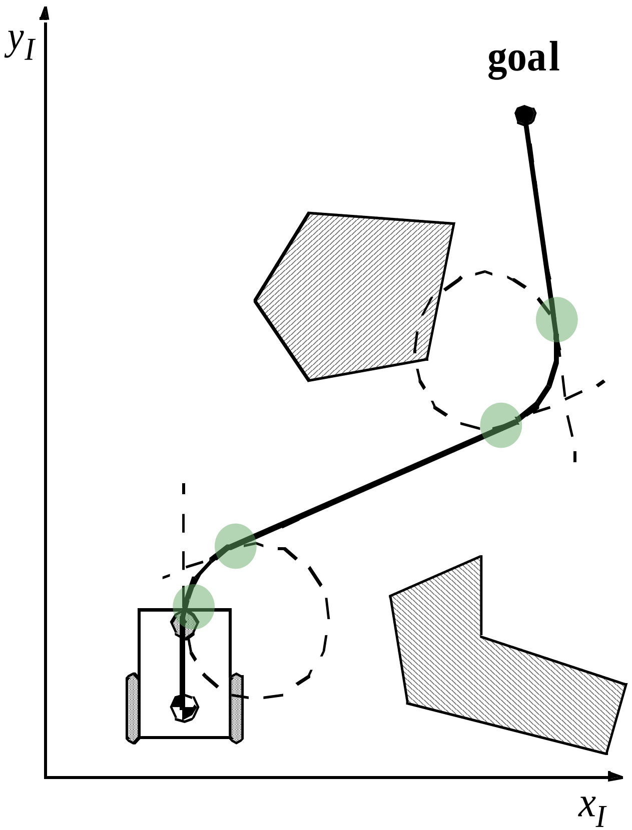
Motion Control
wheeled robots

Wheeled Mobile Robot Motion Control: Overview

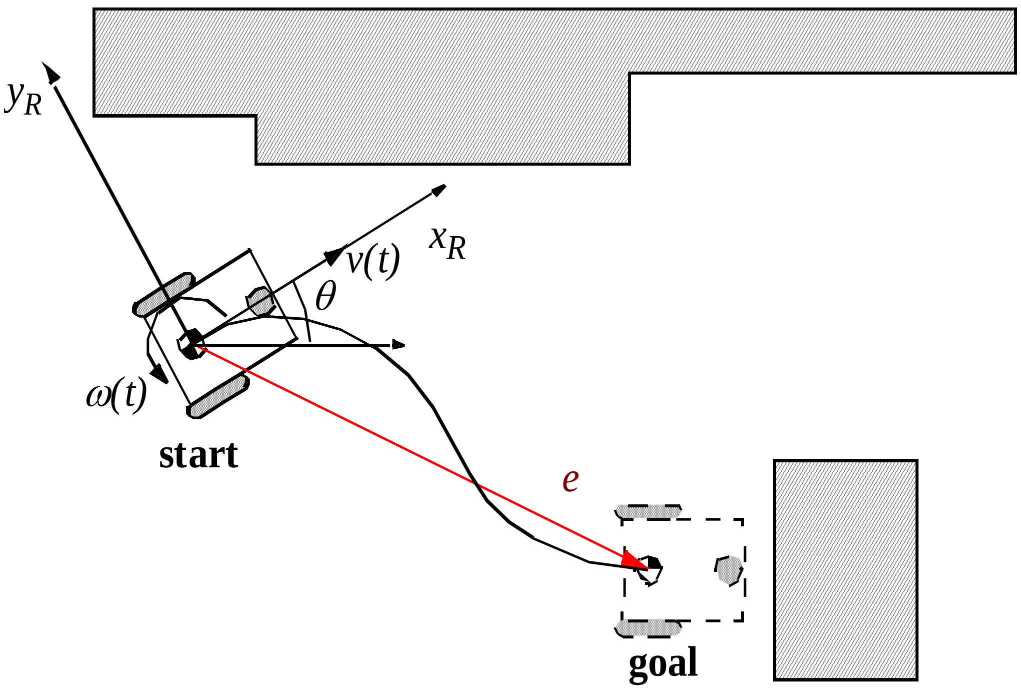
- The objective of a kinematic controller is to follow a trajectory described by its position and/or velocity profiles as function of time.
- Motion control is not straight forward because mobile robots are typically non-holonomic and MIMO systems.
- Most controllers (including the one presented here) are not considering the dynamics of the system

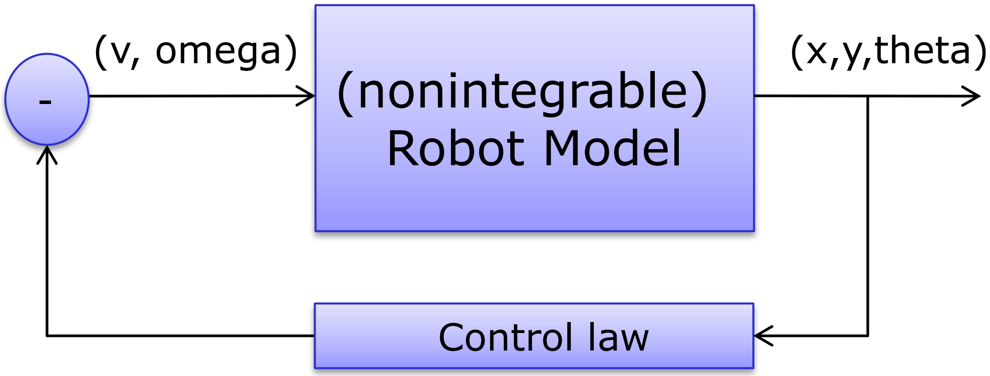
Motion Control: Open Loop Control

- trajectory (path) divided in motion segments of clearly defined shape:
  - straight lines and segments of a circle
  - Dubins car, and Reeds-Shepp car
- control problem:
  - pre-compute a smooth trajectory based on line, circle (and clothoid) segments
- Disadvantages:
  - It is not at all an easy task to pre-compute a feasible trajectory
  - limitations and constraints of the robots velocities and accelerations
  - does not adapt or correct the trajectory if dynamical changes of the environment occur.
  - The resulting trajectories are usually not smooth (in acceleration, jerk, etc.)



Motion Control: Feedback Control





- Find a control matrix *K*, if exists

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix}$$

with $k_{ij}=k(t,e)$

- such that the control of *v(t)* and $\omega(t)$

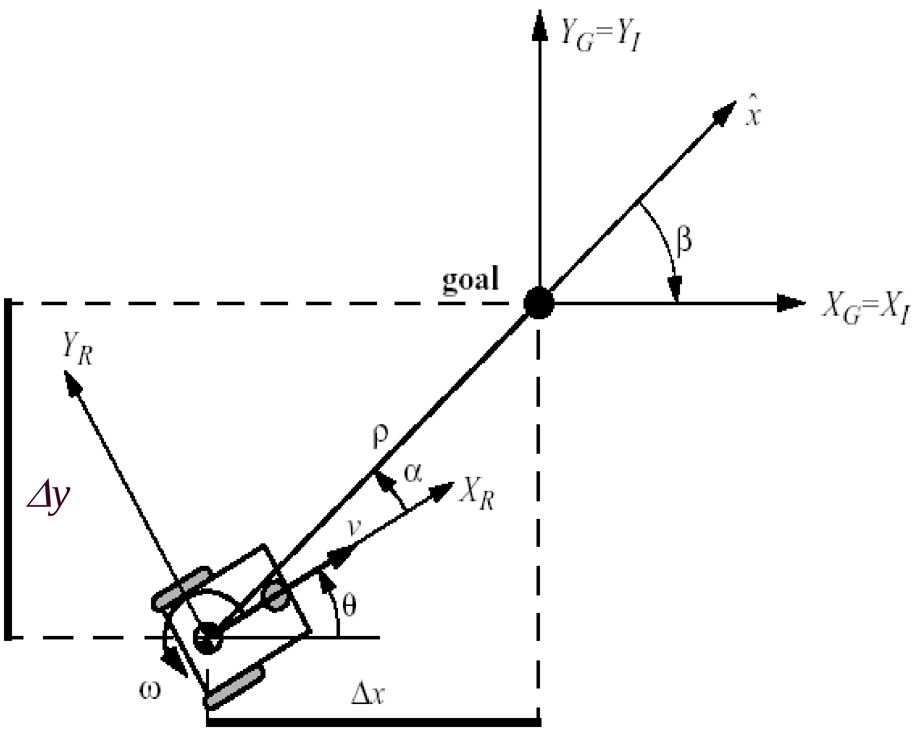
$$\begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = K \cdot e = K \cdot {}^{R}\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

- drives the error e to zero

$$\lim_{t \to \infty} e(t) = 0$$

- MIMO state feedback control

Motion Control: Kinematic Position Control



- The kinematics of a differential drive mobile robot described in the inertial frame $\{x_I, y_I, \theta\}$ is given by,

$$^{I}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

- where $\dot{x}$ and $\dot{y}$ are the linear velocities in the direction of the x_I and y_I of the inertial frame.
- Let alpha denote the angle between the x_R axis of the robots reference frame and the vector connecting the center of the axle of the wheels with the final position.

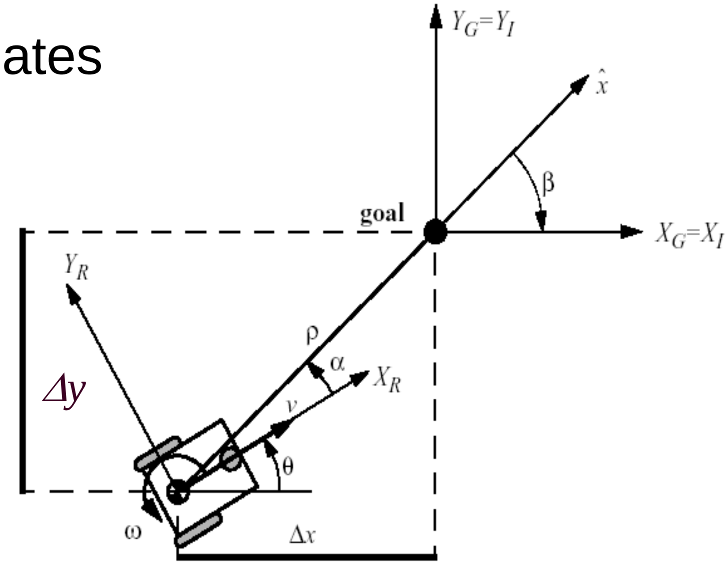
Kinematic Position Control: Coordinates Transformation

- Coordinates transformation into polar coordinates with its origin at goal position:

$$\rho = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\alpha = -\theta + \operatorname{atan2}(\Delta y, \Delta x)$$

$$\beta = -\theta - \alpha$$



- System description, in the new polar coordinates

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\cos\alpha & 0 \\ \frac{\sin\alpha}{\rho} & -1 \\ -\frac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

for $I_1 = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 \\ -\frac{\sin\alpha}{\rho} & -1 \\ \frac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

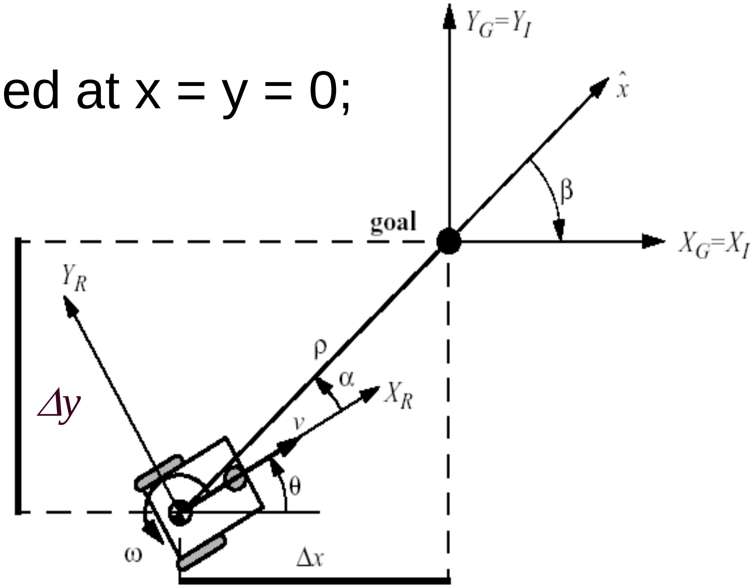
for $I_2 = (-\pi, -\pi/2] \cup (\pi/2, \pi]$

Kinematic Position Control: Remarks

- The coordinates transformation is not defined at x = y = 0;

- For $\alpha \in I_1$ the forward direction of the robot points toward the goal, for $\alpha \in I_2$ it is the backward direction.

$$\alpha \in I_1 = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$$



- By properly defining the forward direction of the robot at its initial configuration, it is always possible to have $\alpha \in I_1$ at t=0. However this does not mean that a remains in I_1 for all time t.

Kinematic Position Control: The Control Law

- It can be shown, that with

$$v = k_\rho \rho \qquad \omega = k_\alpha \alpha + k_\beta \beta$$

  the feedback controlled system

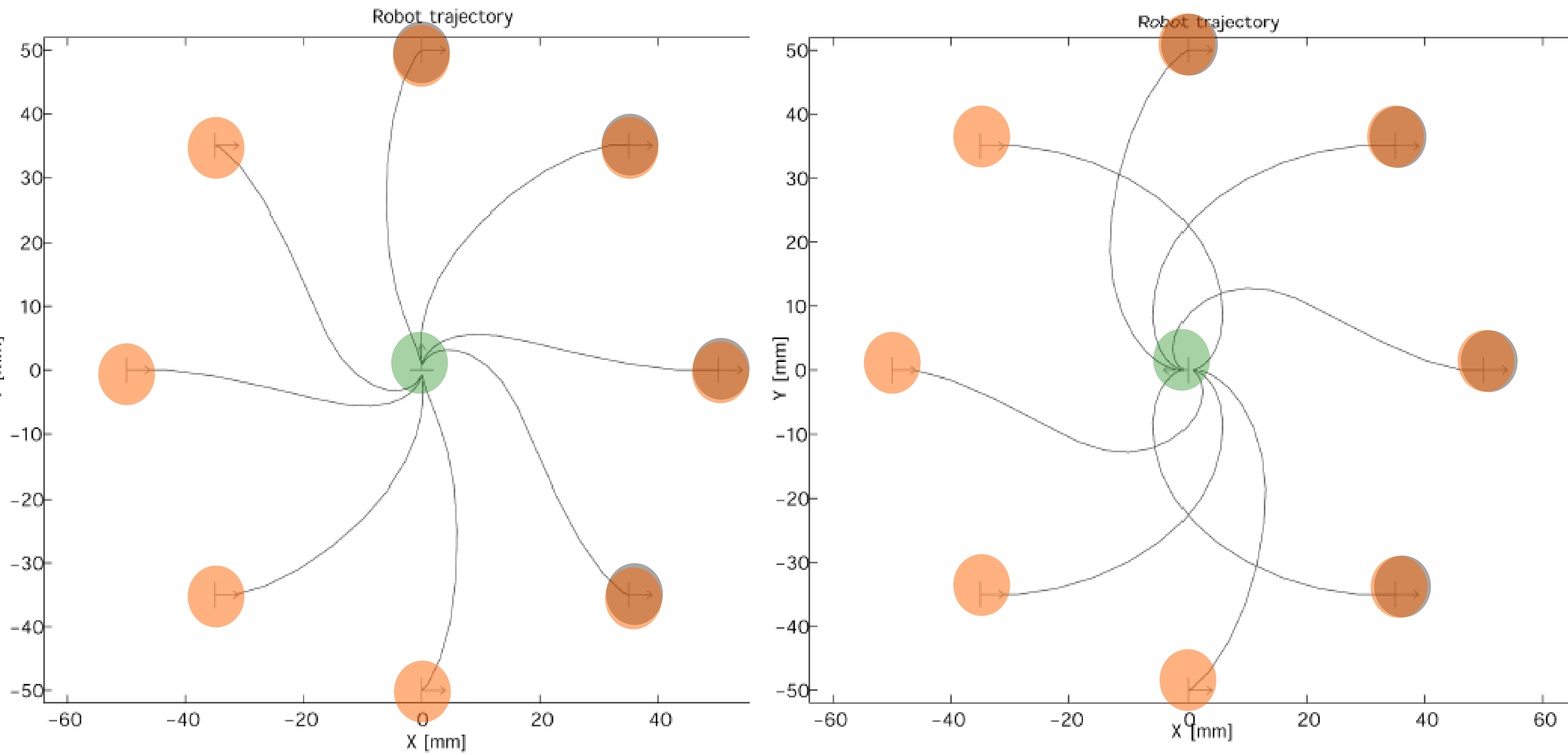
$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -k_\rho \rho \cos\alpha \\ k_\rho \sin\alpha - k_\alpha \alpha - k_\beta \beta \\ -k_\rho \sin\alpha \end{bmatrix}$$

  will drive the robot to $(\rho,\alpha,\beta) = (0,0,0)$

- The control signal v has always constant sign,
  - the direction of movement is kept positive or negative during movement
  - parking maneuver is performed always in the most natural way and without ever inverting its motion.

Kinematic Position Control: Resulting Path

- The goal is in the center and the initial position on the circle.



$$k = (k_\rho, k_\alpha, k_\beta) = (3, 8, -1.5)$$

43 Kinematic Position Control: Stability Issue

- It can further be shown, that the closed loop control system is locally exponentially stable if

$$k_\rho > 0 \;\; ; \;\; k_\beta < 0 \;\; ; \;\; k_\alpha - k_\rho > 0$$

$$\mathrm{k} = (\mathrm{k}_\rho, \mathrm{k}_\alpha, \mathrm{k}_\beta) = (3,8,-1.5)$$

- Proof:
  for small x –> cos$x$ = 1, sin$x$ = x

$$\begin{bmatrix} \dot\rho \\ \dot\alpha \\ \dot\beta \end{bmatrix} = \begin{bmatrix} -k_\rho & 0 & 0 \\ 0 & -(k_\alpha - k_\rho) & -k_\beta \\ 0 & -k_\rho & 0 \end{bmatrix} \begin{bmatrix} \rho \\ \alpha \\ \beta \end{bmatrix} \qquad A = \begin{bmatrix} -k_\rho & 0 & 0 \\ 0 & -(k_\alpha - k_\rho) & -k_\beta \\ 0 & -k_\rho & 0 \end{bmatrix}$$

  and the characteristic polynomial of the matrix A of all roots

$$(\lambda + k_\rho)(\lambda^2 + \lambda(k_\alpha - k_\rho) - k_\rho k_\beta)$$

  have negative real parts.