ELEC4410

Control System Design

Lecture 19: Feedback from Estimated States and Discrete-Time Control Design

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Outline

- Feedback from Estimated States
- Discrete-Time Control Design
- Dead-Beat Control

Feedback from Estimated States

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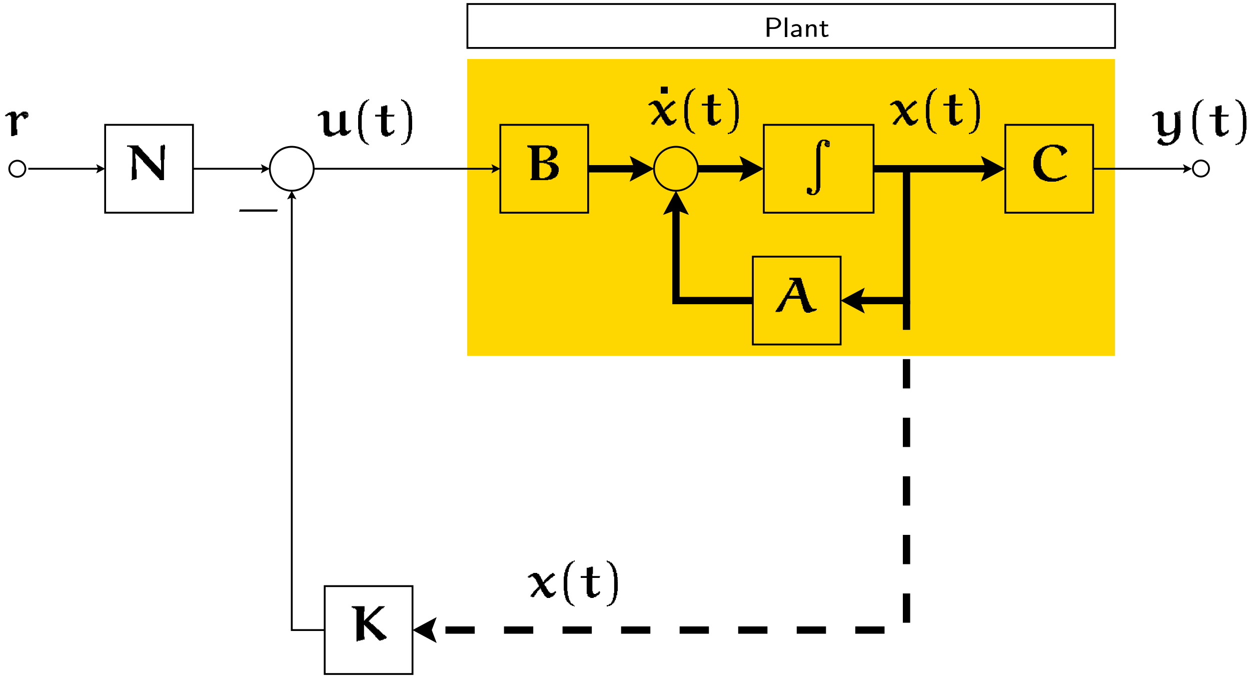
- a **state feedback gain** K



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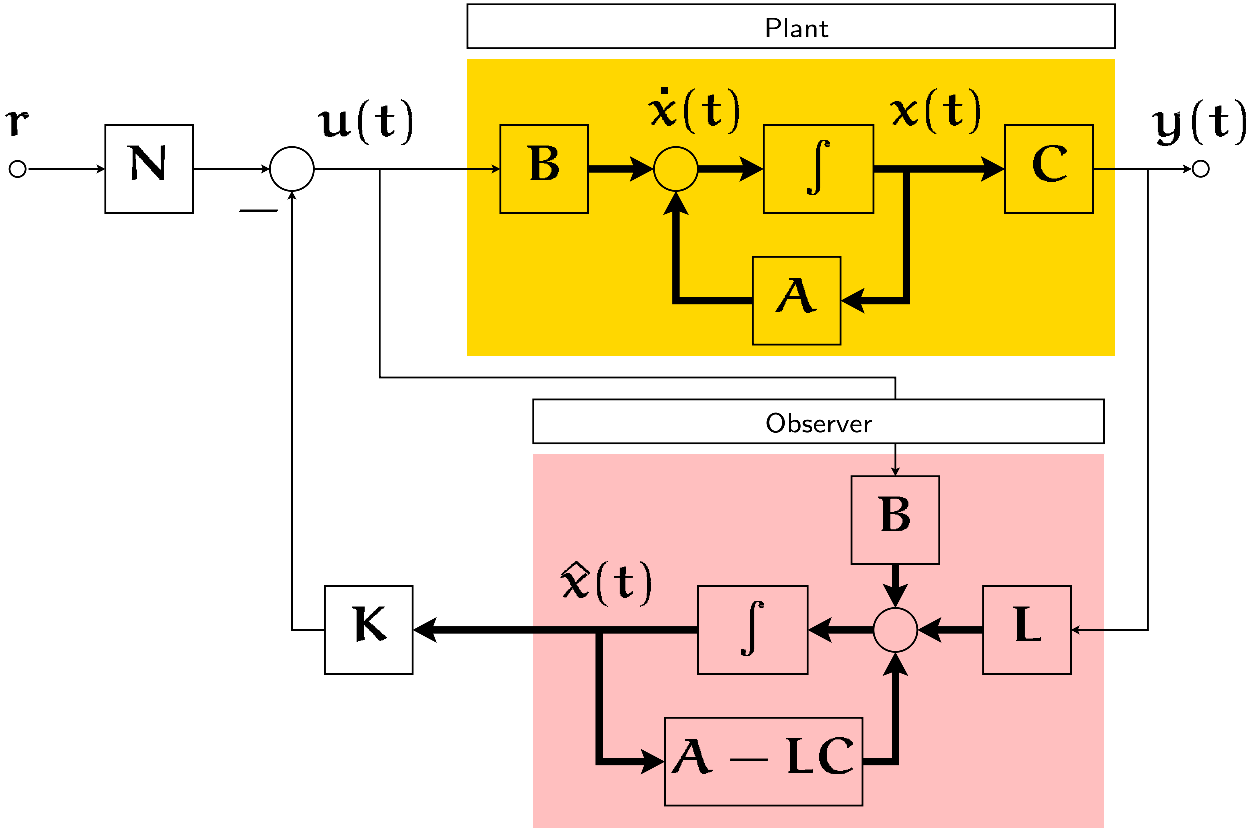
- a **state feedback gain** $\mathbf{K}$
- a **state estimator** (observer), with gain $\mathbf{L}$



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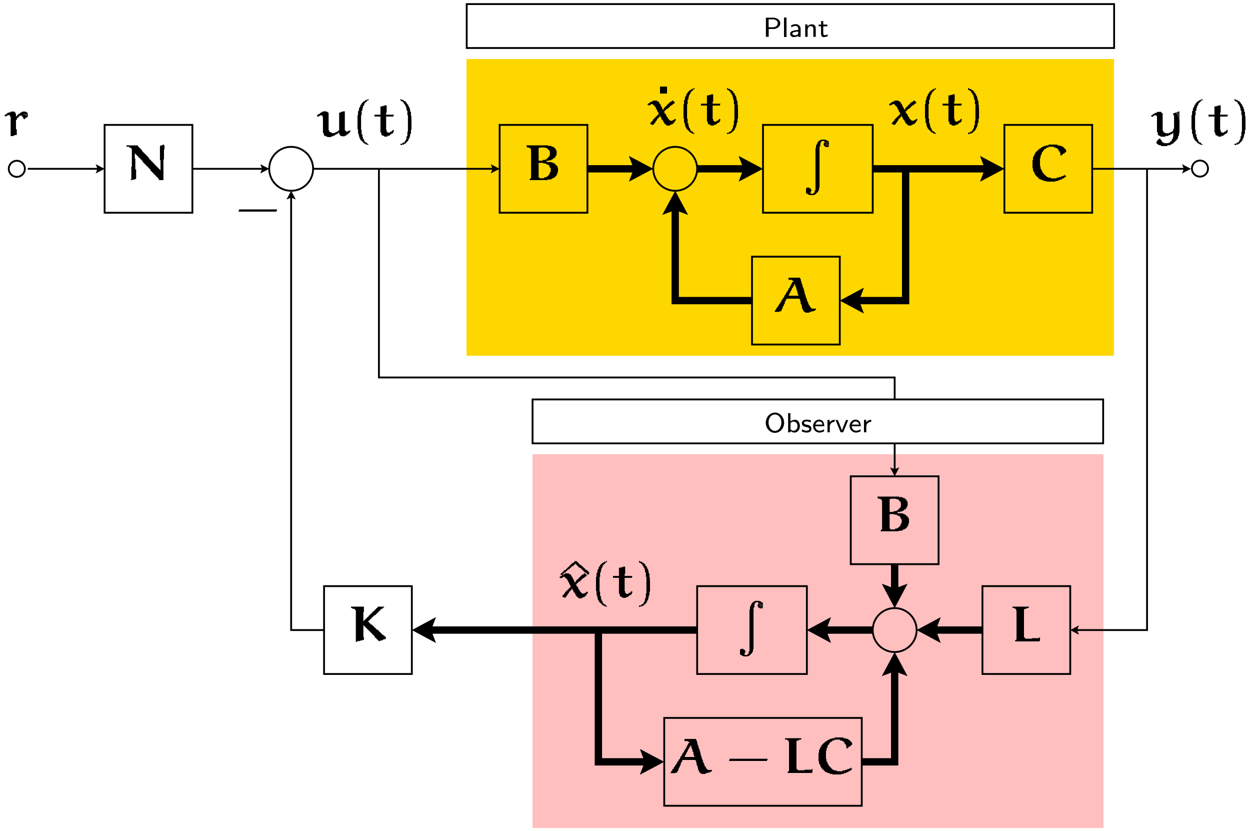
- a **state estimator** (observer), with gain $\mathbf{L}$

To design $\mathbf{K}$ we used the **Bass-Gura** formula to make

$$(\mathbf{A} - \mathbf{BK}): \quad \text{Hurwitz}$$

To design $\mathbf{L}$ in the observer, we used **duality** and the same **Bass-Gura** formula to make

$$(\mathbf{A} - \mathbf{LC}): \quad \text{Hurwitz}$$



Feedback from Estimated States

- By designing $\mathbf{K}$ such that $(\boldsymbol{A} - \mathbf{B}\mathbf{K})$ is Hurwitz, with the desired eigenvalues, we can guarantee that the closed-loop system will be asymptotically and BIBO stable, and will have the specified dynamic response.

Feedback from Estimated States

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Feedback from Estimated States

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- By designing $\mathbf{L}$ such that $(\mathbf{A}-\mathbf{LC})$ is Hurwitz, we guarantee that the observer will be asymptotically stable, and the estimate of the states $\hat{x}(t)$ will converge to the real states $x(t)$ as $t \to \infty$.
- But $\mathbf{K}$ and the observer are designed independently . . . Will they work the same when we put them together in a **feedback from estimated states** scheme?

Feedback from Estimated States

Three basic questions arise regarding **feedback from estimated states:**

- The closed-loop eigenvalues were set as those of $\boxed{(\mathbf{A}-\mathbf{B}\mathbf{K})}$ by using **state feedback**

$$\mathbf{u} = -\mathbf{K}x.$$

Would we still have the same eigenvalues if we do **feedback from estimated states**

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  $$\mathbf{u}=-\mathbf{K}\hat{x}\,?$$

- Would the interconnection affect the **observer** eigenvalues, those of $\boxed{(\mathbf{A}-\mathbf{LC})}$?
- What would be the effect of the observer in the **closed-loop transfer function**?

Feedback from Estimated States

To answer these questions we take a look at the state equations of the full system, putting together plant and observer, that is,

$$\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x}+\mathbf{B}\mathbf{u} && \text{plant}\\
\dot{\hat{\mathbf{x}}} &= (\mathbf{A}-\mathbf{L}\mathbf{C})\hat{\mathbf{x}}+\mathbf{L}\mathbf{C}\mathbf{x}+\mathbf{B}\mathbf{u} && \text{observer}
\end{aligned}$$

Feedback from Estimated States

To answer these questions we take a look at the state equations of the full system, putting together plant and observer, that is,

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{K}\hat{\mathbf{x}} + \mathbf{B}\mathbf{N}\mathbf{r} && \text{plant} \\ \dot{\hat{\mathbf{x}}} &= (\mathbf{A} - \mathbf{L}\mathbf{C})\hat{\mathbf{x}} + \mathbf{L}\mathbf{C}\mathbf{x} - \mathbf{B}\mathbf{K}\hat{\mathbf{x}} + \mathbf{B}\mathbf{N}\mathbf{r} && \text{observer} \end{aligned}$$

after replacing $\mathbf{u} = \mathbf{N}\mathbf{r} - \mathbf{K}\hat{\mathbf{x}}$.

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$$\dot{\hat{\mathbf{x}}} = (\mathbf{A} - \mathbf{L}\mathbf{C})\hat{\mathbf{x}} + \mathbf{L}\mathbf{C}\mathbf{x} - \mathbf{B}\mathbf{K}\hat{\mathbf{x}} + \mathbf{B}\mathbf{N}\mathbf{r} \qquad \text{observer}$$

after replacing $\mathbf{u} = \mathbf{N}\mathbf{r} - \mathbf{K}\hat{\mathbf{x}}$. Packaging these equations in a more compact form we have

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\hat{\mathbf{x}}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & -\mathbf{B}\mathbf{K} \\ \mathbf{L}\mathbf{C} & (\mathbf{A} - \mathbf{L}\mathbf{C} - \mathbf{B}\mathbf{K}) \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \hat{\mathbf{x}} \end{bmatrix} + \begin{bmatrix} \mathbf{B}\mathbf{N} \\ \mathbf{B}\mathbf{N} \end{bmatrix} \mathbf{r}$$

$$\mathbf{y} = \begin{bmatrix} \mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \hat{\mathbf{x}} \end{bmatrix}$$

Feedback from Estimated States

Let's make a change of coordinates, so that the new coordinates are **the plant state** x and **the estimation error** $\varepsilon = x - \hat{x}$,

$$\begin{bmatrix} x \\ \varepsilon \end{bmatrix} = \begin{bmatrix} x \\ x - \hat{x} \end{bmatrix} = \underbrace{\begin{bmatrix} I & 0 \\ I & -I \end{bmatrix}}_{P} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

Note that $P^{-1} = \left[\begin{smallmatrix} I & 0 \\ I & -I \end{smallmatrix}\right]^{-1} = \left[\begin{smallmatrix} I & 0 \\ I & -I \end{smallmatrix}\right] = P$.

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Note that $P^{-1} = \left[\begin{smallmatrix} I & 0 \\ I & -I \end{smallmatrix}\right]^{-1} = \left[\begin{smallmatrix} I & 0 \\ I & -I \end{smallmatrix}\right] = P$. With this equivalence transformation we get the matrices in the new coordinates as

$$\bar{A}_{KL} = PA_{KL}P^{-1} = \begin{bmatrix} (A - BK) & BK \\ 0 & (A - LC) \end{bmatrix}, \quad \bar{B}_{KL} = PB_{KL} = \begin{bmatrix} BN \\ 0 \end{bmatrix}$$

$$\bar{C}_{KL} = C_{KL}P^{-1} = \begin{bmatrix} C & 0 \end{bmatrix}$$

Feedback from Estimated States

The full system in the new coordinates is thus represented as

$$\begin{bmatrix} \dot{x} \\ \dot{\varepsilon} \end{bmatrix} = \begin{bmatrix} (A - BK) & BK \\ 0 & (A - LC) \end{bmatrix} \begin{bmatrix} x \\ \varepsilon \end{bmatrix} + \begin{bmatrix} BN \\ 0 \end{bmatrix} r$$

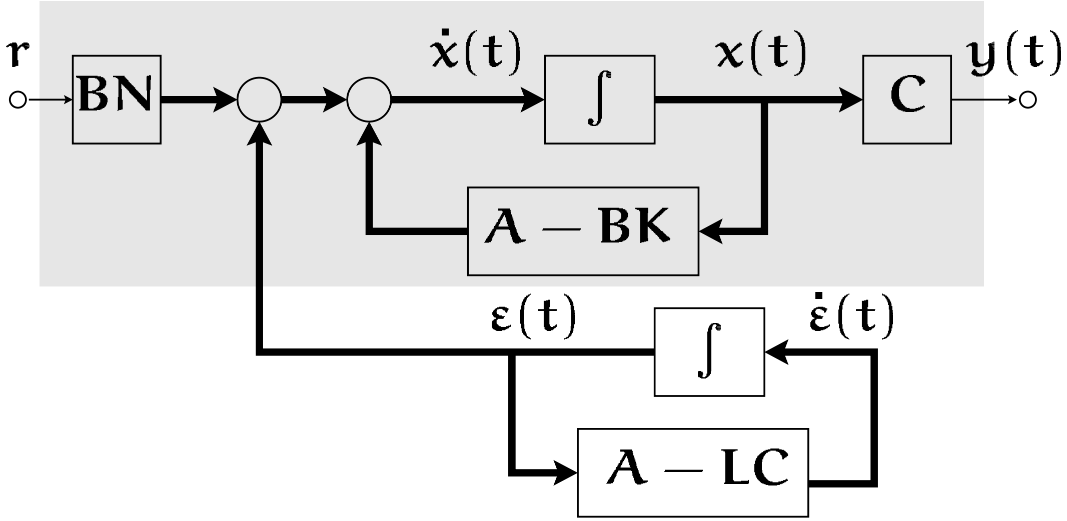
$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ \varepsilon \end{bmatrix}$$

Because $\bar{A}_{KL}$ is **triangular**, its eigenvalues are the union of those of $(A - BK)$ and $(A - LC)$.

Controller and observer **do not affect each other** in the interconnection.

Feedback from Estimated States

Note that the estimation error is **uncontrollable**, hence the observer eigenvalues will not appear in the closed loop transfer function.



Feedback from Estimated States

The property of independence between control and state estimation is called the **Separation Principle**

Separation Principle: The design of the state feedback and the design of the state estimator can be carried out independently.

- The eigenvalues of the closed-loop system are as designed by the state feedback law, unaffected by the use of a state estimator.

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Separation Principle: The design of the state feedback and the design of the state estimator can be carried out independently.

- The eigenvalues of the closed-loop system are as designed by the state feedback law, unaffected by the use of a state estimator.
- The eigenvalues of the observer are unaffected by the state feedback law.

Feedback from Estimated States

The closed-loop transfer function will only have the eigenvalues arising from $(\mathbf{A} - \mathbf{BK})$, since the estimation error is **uncontrollable**,

$$\mathbf{G_{cl}}(\mathbf{s}) = \mathbf{C}(\mathbf{sI} - \mathbf{A} + \mathbf{BK})^{-1}\mathbf{BN}.$$

Transients in state estimation, however, will be seen at the output, since the estimation error is **observable**.

Outline

- Feedback from Estimated States
- Discrete-Time Control Design
- Dead-Beat Control

Discrete-Time Control Design

For discrete-time state equations

$$\boxed{\mathbf{x}[k+1] = \mathbf{A}\mathbf{x}[k] + \mathbf{B}\mathbf{u}[k], \qquad \mathbf{y}[k] = \mathbf{C}\mathbf{x}[k],}$$

the **design procedure** for a state feedback law $\mathbf{u}[k] = -\mathbf{K}\mathbf{x}[k]$ is **the same as for continuous-time systems**.

The same goes for a discrete-time state observer,

$$\boxed{\hat{\mathbf{x}}[k+1] = (\mathbf{A} - \mathbf{L}\mathbf{C})\hat{\mathbf{x}}[k] + \mathbf{B}\mathbf{u}[k] + \mathbf{L}\mathbf{y}[k].}$$

One difference is the **location of the desired eigenvalues**. E.g., for asymptotic stability, they should be **inside the unit circle**.

Discrete-Time Control Design

For discrete-time state equations

$$\boxed{x[k+1] = Ax[k] + Bu[k], \qquad y[k] = Cx[k],}$$

the **design procedure** for a state feedback law $u[k] = -Kx[k]$ is **the same as for continuous-time systems**.

The same goes for a discrete-time state observer,

$$\boxed{\hat{x}[k+1] = (A - LC)\hat{x}[k] + Bu[k] + Ly[k].}$$

One difference is the **location of the desired eigenvalues**. E.g., for asymptotic stability, they should be **inside the unit circle**.

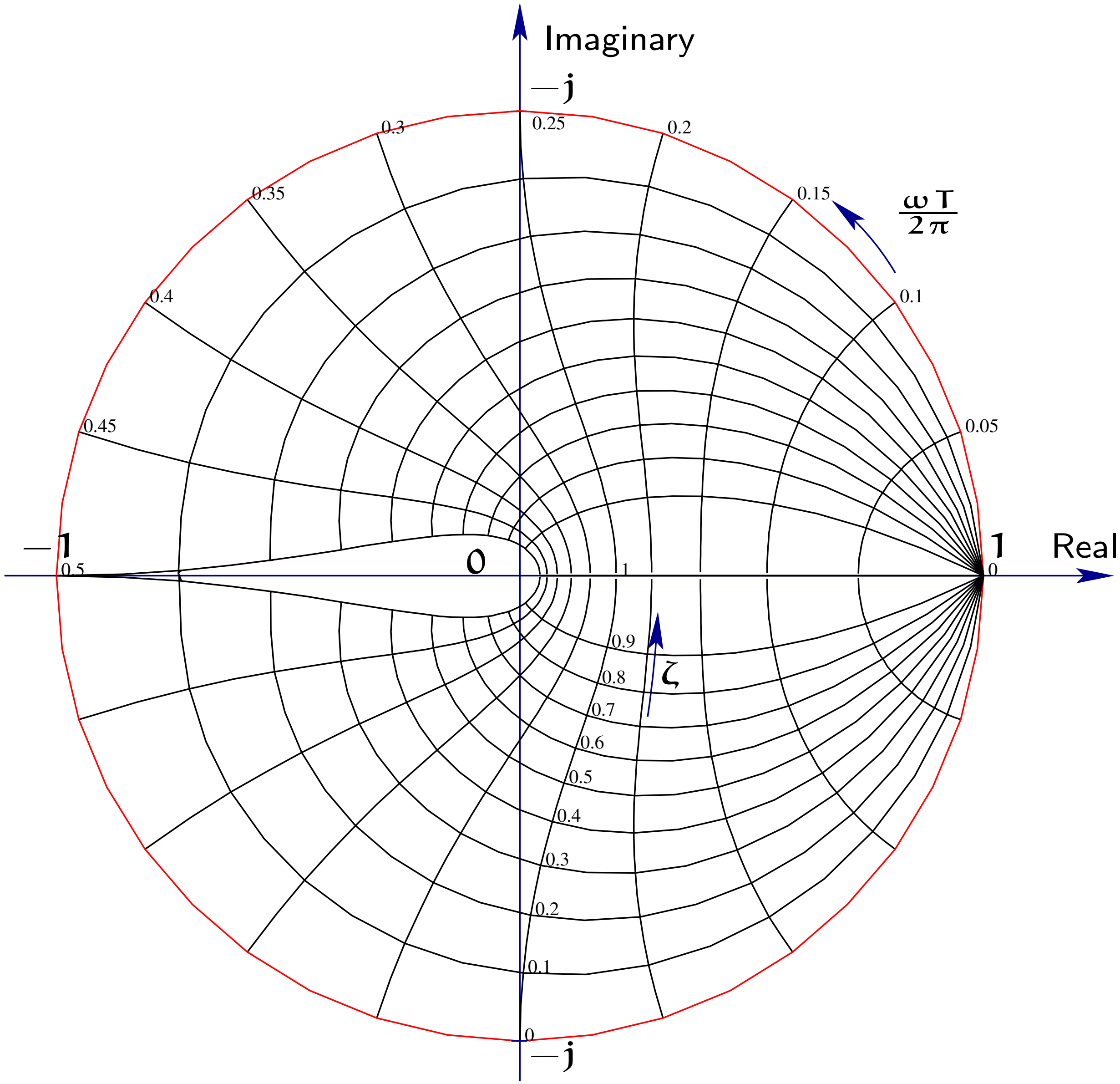
A practical rule to choose desired discrete-time eigenvalues is

1. choose a desired location in **continuous-time**, say p_i
2. translate it to discrete-time using the relation $\boxed{\lambda_i = e^{p_i T}}$.

Discrete-Time Control Design

Loci with constant damping ζ and constant frequencies ω in the discrete complex plane.

In MATLAB this grid may be obtained with the `zgrid` command.



Discrete-Time Control Design: Example

Example (Discrete-time speed control of a DC motor). We return to the DC motor we considered in the examples of the last lecture. We will suppose that the motor is to be controlled with a PC. Hence, the controller has to be **discrete-time**.

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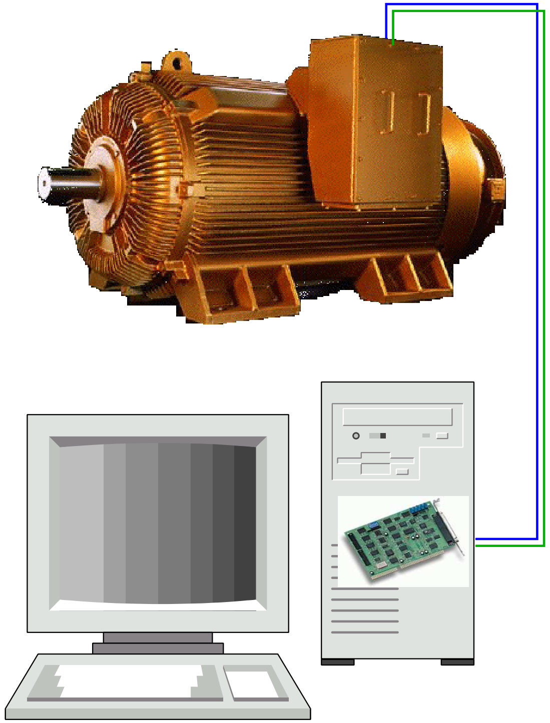
This time, with a view to design a discrete-time control law, we first **discretise** the continuous-time model

$$\frac{d}{dt}\begin{bmatrix}\omega(t)\\ i(t)\end{bmatrix} = \begin{bmatrix}-10 & 1\\ -0.02 & -2\end{bmatrix}\begin{bmatrix}\omega(t)\\ i(t)\end{bmatrix} + \begin{bmatrix}0\\ 2\end{bmatrix} V(t)$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\omega(t)\\ i(t)\end{bmatrix}$$



The first **design parameter** to define is the **sampling period** T.

Discrete-Time Control Design: Example

Example (Discrete design, step 1: choice of sampling period). From **Shannon's Sampling Theorem**, the sampling frequency $\omega_s = 2\pi/T$ should be **at least twice the bandwidth of the closed-loop system** (because we will **change** the system bandwidth with the control action).

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The specification that we had for the previous continuous-time design was a settling time t_s of about 1s. The rule based on Shannon´s Theorem would then give a sampling time of less than $T = 0.5$s. In practice T is chosen at least 10 to 20 times faster than the desired closed-loop settling time. Here we choose

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Note that the maximum sampling speed is limited by the available computer clock frequency, and thus the time required for computations and signal processing operations.

Discrete-Time Control Design: Example

Example (Discrete design, step 2: discretisation of the plant). Having defined the sampling-time T, we compute the discrete-time state matrices

$$A_d = e^{At}, \quad \text{and} \quad B_d = \int_0^T e^{A\tau} B \, d\tau$$

From MATLAB, we do `[Ad,Bd]=c2d(A,B,0.1)` and obtain

$$A_d = \begin{bmatrix} 0.3678 & 0.0563 \\ -0.0011 & 0.8186 \end{bmatrix}, \quad B_d = \begin{bmatrix} 0.0068 \\ 0.1812 \end{bmatrix}$$

As we can verify, the open loop **discrete-time** eigenvalues are

$$0.3679 = e^{(-9.9975 \times 0.1)} \quad \text{and} \quad 0.8185 = e^{(-2.0025 \times 0.1)}$$

Discrete-Time Control Design: Example

Example (Discrete design, step 3: design of discrete feedback gain). The **discrete-time** characteristic polynomial is

$$\Delta(z) = (z - e^{-9.9975\mathrm{T}})(z - e^{-2.0025\mathrm{T}}) = z^2 - 1.1864z + 0.3011$$

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For the desired discrete characteristic polynomial, we first obtain the **discrete mapping** $p \to e^{pT}$ of the eigenvalues $p_{1,2} = -5 \pm j$ specified for the continuous-time system, i.e.,

$$\delta \pm j\gamma = e^{(-5.\;\;j).\;\;0.1} = 0.6035 \pm j0.0605$$

which yield the desired discrete characteristic polynomial

$$\Delta_K(z) = z^2 - 1.2070z + 0.3678.$$

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which yield the desired discrete characteristic polynomial

$$\Delta_K(z) = z^2 - 1.2070z + 0.3678.$$

From the coefficients of $\Delta_K(z)$ and $\Delta(z)$ we get the discrete state feedback gain

$$\bar{K}_d = \begin{bmatrix}(-1.2070 + 1.1864) & (0.3678 - 0.3011)\end{bmatrix} = \begin{bmatrix}-0.0206 & 0.0667\end{bmatrix}$$

Discrete-Time Control Design: Example

Example (Discrete design, step 3, continuation). We then compute, just following the same procedure used in continuous-time, from the **discrete** matrices $\mathbf{A_d}$ and $\mathbf{B_d}$, the **discrete controllability matrices** $\mathcal{C}_\mathbf{d}$ and $\bar{\mathcal{C}}_\mathbf{d}$,

$$\mathcal{C}_\mathbf{d} = \begin{bmatrix} 0.0068555 & 0.0127368 \\ 0.1812645 & 0.1483875 \end{bmatrix}, \quad \text{and} \quad \bar{\mathcal{C}}_\mathbf{d} = \begin{bmatrix} 1 & -1.1864 \\ 0 & 1 \end{bmatrix}^{-1}$$

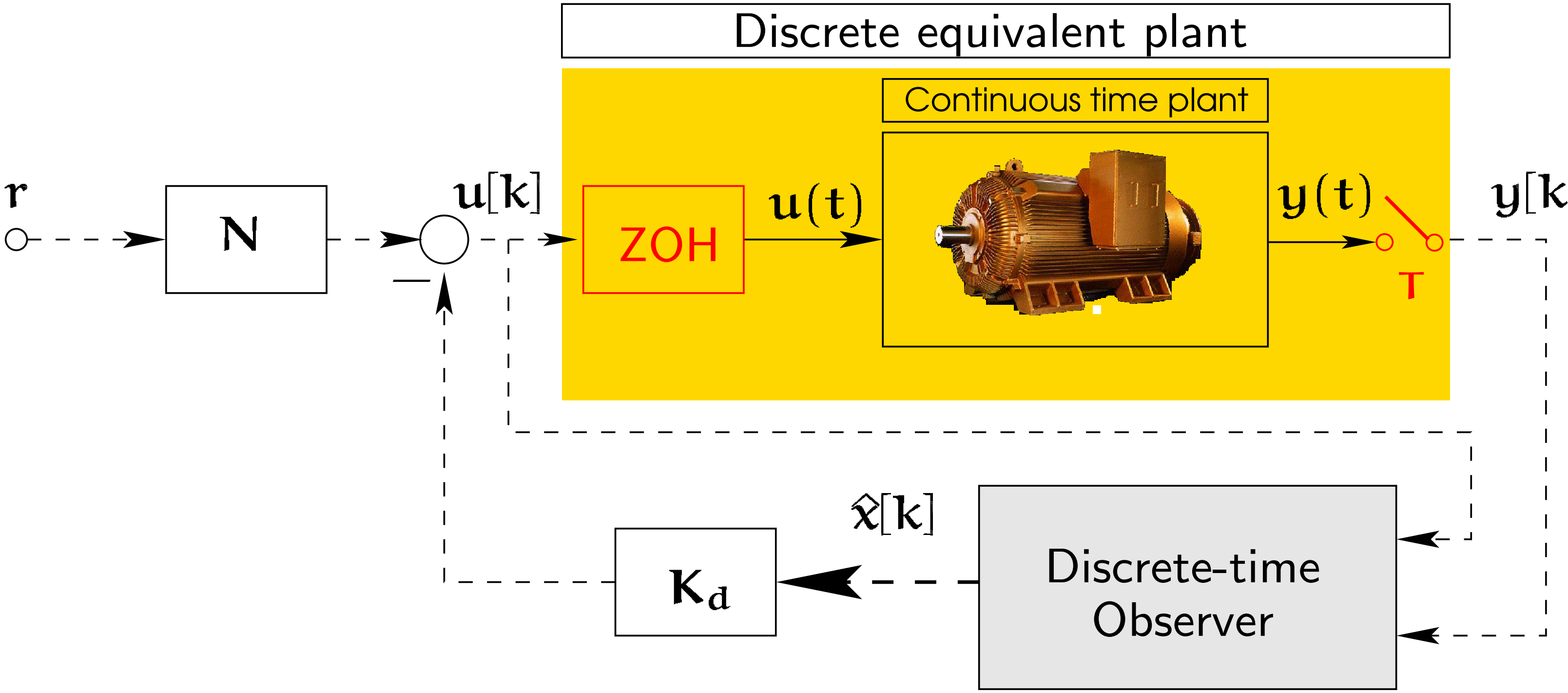
We thus obtain, in the original coordinates, the **discrete-time** feedback gain

$$K_\mathbf{d} = \bar{K}_\mathbf{d} \bar{\mathcal{C}}_\mathbf{d} \mathcal{C}_\mathbf{d}^{-1} = \begin{bmatrix} 8.3011164 & -0.4270676 \end{bmatrix}$$

As can be verified with MATLAB, $(\mathbf{A_d} - \mathbf{B_d} K_\mathbf{d})$ will have the desired discrete eigenvalues.

Discrete-Time Control Design: Example

Example (Discrete design: continuation). In a similar fashion, we can carry out the design of the discrete-time observer, based on the discrete-time model of the plant. The output feedback design is finally implemented on the **continuous-time** plant through a Zero Order Hold and a Sampler.



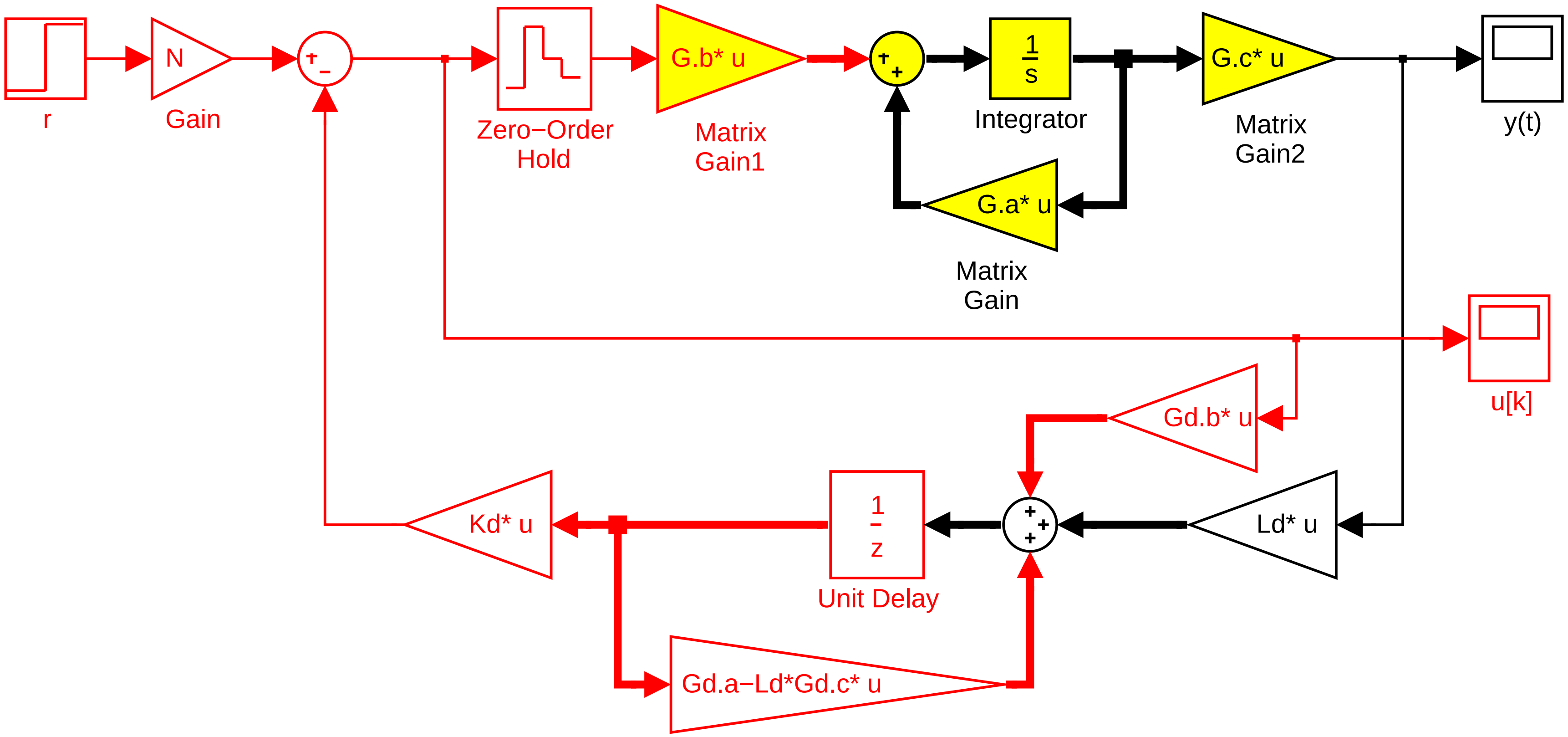
Discrete-Time Control Design: Example

Example (Discrete design: MATLAB script). We used the following MATLAB script to compute the gains and run the simulations.

```
% Continuous-time matrices
A=[-10 1;-0.02 -2];B=[0;2];C=[1 0];D=0;
G=ss(a,B,C,D);    % state space system definition
T=0.1;            % Sampling time
Gd=c2d(G,T,’zoh’) % discretisation
% Discrete-time feedback gain
Kd=place(Gd.a,Gd.b,exp([-5-i,-5+i]*T))
% Discrete-time observer gain
Ld=place(Gd.a’,Gd.c’,exp([-6-i,-6+i]*T))’
% steady state error compensation
N=inv(Gd.c*inv(eye(2)-Gd.a+Gd.b*Kd)*Gd.b)
% Run simulink diagram
sim(’dmotorOFBK’)
% Plots (after simulations have been run)
subplot(211),plot(y(:,1),y(:,2));grid
subplot(212),stairs(u(:,1),u(:,2));grid
```

Discrete-Time Control Design

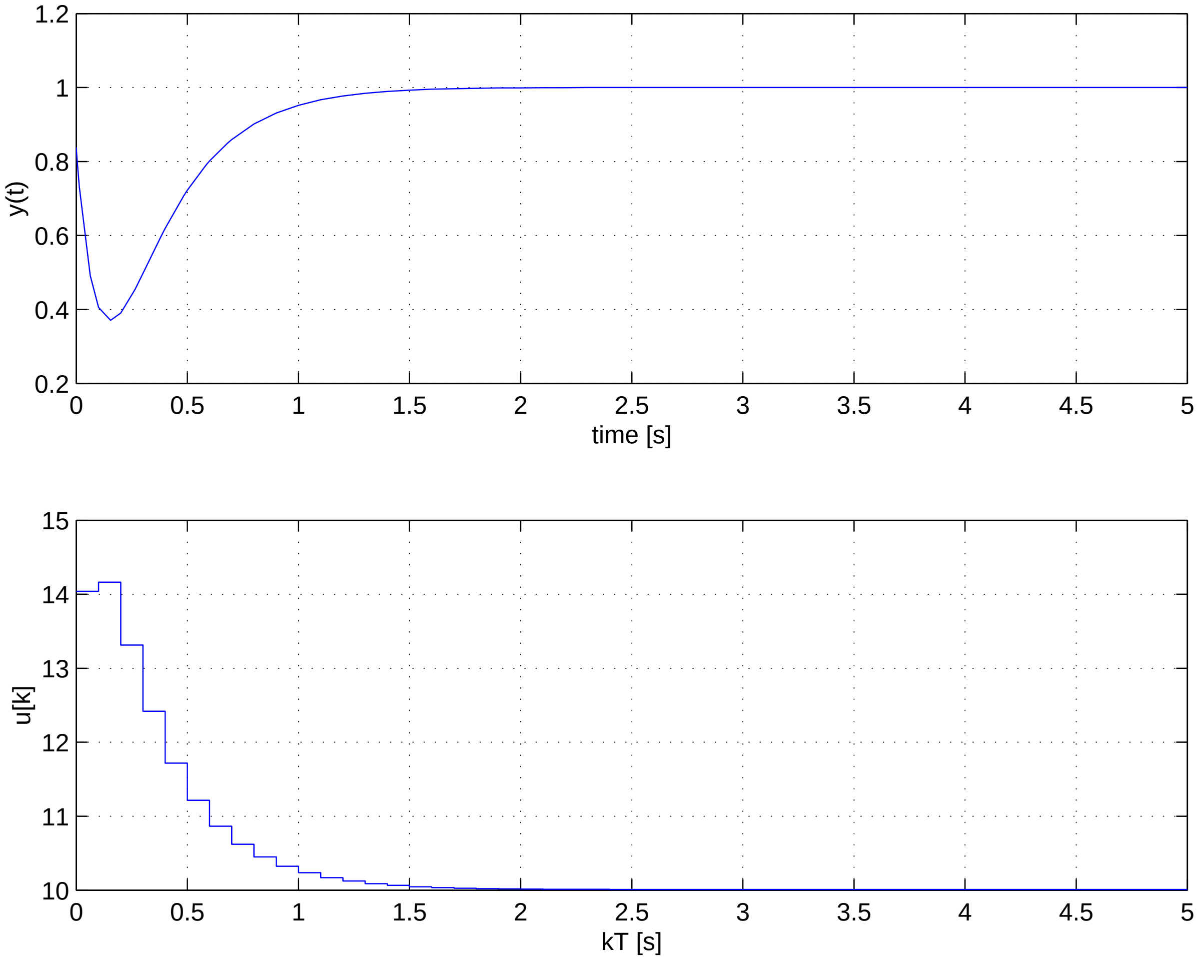
Example (Discrete design: SIMULINK diagram). We used the following SIMULINK diagram to run the simulations.



Notice the **sampled** signals coloured in red (check the option "Sample Time Colors" in the *Format* menu). All blocks with discrete-time signals include a sampler at their input.

Discrete-Time Control Design: Example

Example (Discrete design: simulations). The following plots show the response of the closed-loop sampled-data controlled system: continuous-time output $\mathbf{y}(\mathbf{t})$ and discrete-time control signal $\mathbf{u}[\mathbf{k}]$.



Discrete-Time Design: Peculiarities

Two $\boxed{\textbf{special differences}}$ in the discrete-time design procedure:

- The gain $\mathbf{N}$ for steady state error compensation is, as in continuous-time, the inverse of the steady-state DC gain of the **closed-loop** transfer function. **Notice** though, that in discrete-time

$$\mathbf{N} = \frac{1}{\mathbf{C}(z\mathbf{I} - \mathbf{A_d} + \mathbf{B_d K_d})^{-1}\mathbf{B_d}\Big|_{z=1}} = \frac{1}{\mathbf{C}(\mathbf{I} - \mathbf{A_d} + \mathbf{B_d K_d})^{-1}\mathbf{B_d}}$$

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- If we implement robust tracking by adding **Integral Action** into the state feedback design, notice that the plant augmentation is different in discrete-time,

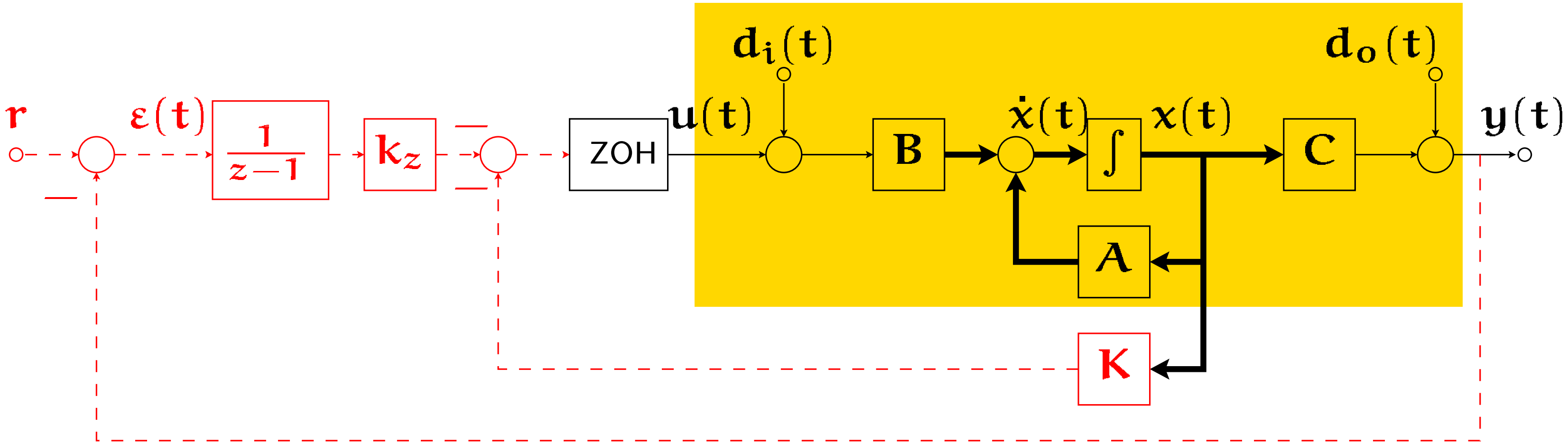
$$\mathbf{A_a} = \begin{bmatrix} \mathbf{A_d} & 0 \\ -\mathbf{C} & 1 \end{bmatrix}$$

since the **discrete-integration** of the tracking error has to be implemented as

$$z[k+1] = z[k] + r - \mathbf{C}x[k]$$

Discrete-Time Design: Peculiarities

The implementation of the **discrete-time integral action** in the diagram should be consistent, i.e. **discrete-integration** of the tracking error.



Apart from these two differences, and the locations for the eigenvalues/poles, the discrete-time design is obtained by performing the same computations for continuous-time state feedback and observer design.

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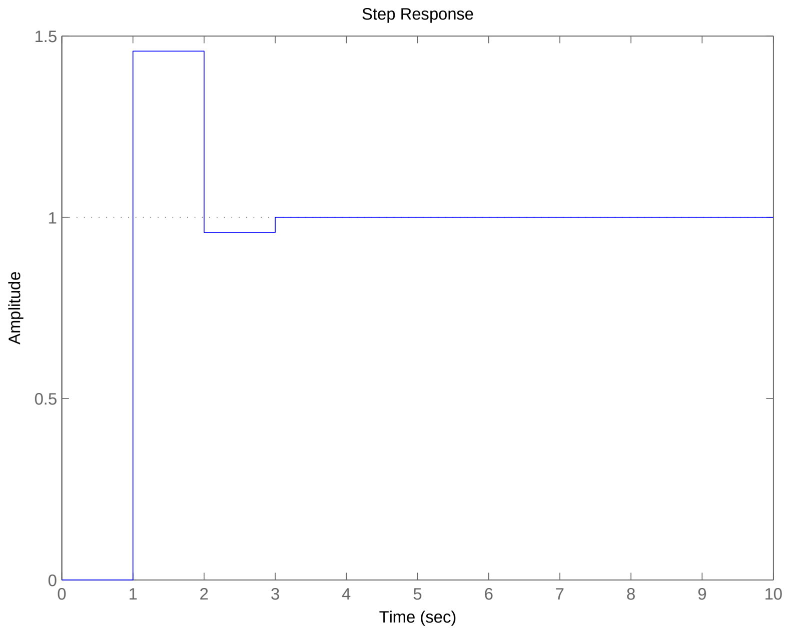
Discrete-Time Design: Dead-Beat Control

A special discrete-time design that **has no correlate** in continuous-time is **dead-beat control**.

A **dead-beat** response is a response that settles in its final value at a **finite time**. This feature arises in a discrete-time system which has all its poles at $z = 0$, e.g.,

$$G(z) = \frac{(7z - 1)(5z - 1)}{24z^3},$$



which will settle in 3 sampling periods (the dynamics is just a **delay** of 3 sample periods).

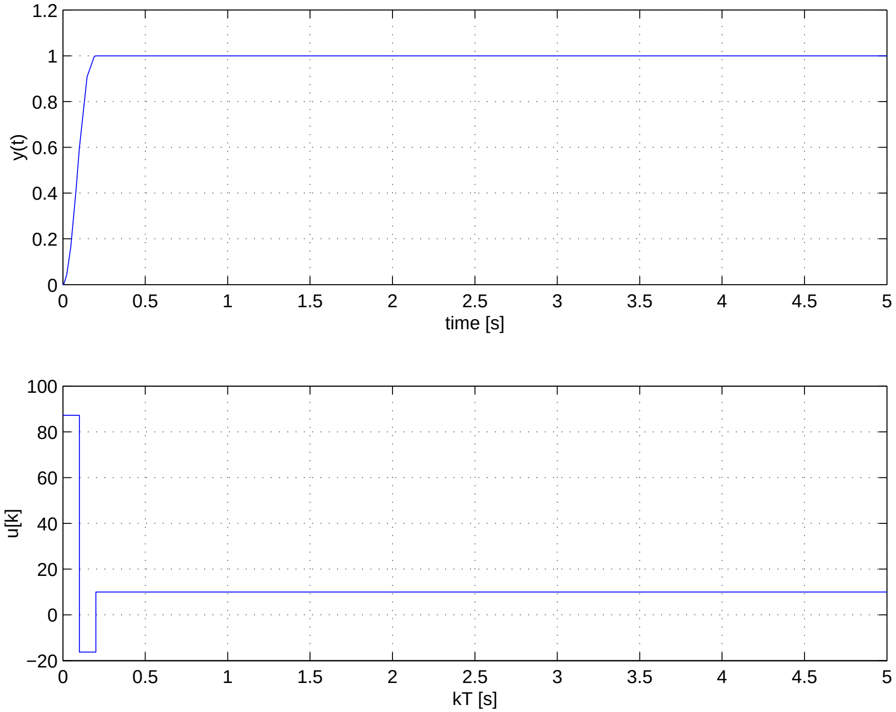
Discrete-Time Design: Dead-Beat Control

To design a **dead-beat** controller, we just have to find $\mathbf{K_d}$ to place all the closed-loop poles at $z = 0$. The discrete-time observer can also be designed **dead-beat**, with $\mathbf{L_d}$ to place all the observer poles at $z = 0$.

The plot shows the response of the DC motor of the example controlled to have **dead-beat** response (state feedback — no observer).

There is not much flexibility in dead-beat control, the only parameter to change the response is the sampling period.

Dead-beat usually requires **large control action**, which may saturate actuators.



Conclusions

- We have reviewed the **state feedback** and **state estimator** design procedure, and showed that the **Separation Principle** holds: the two designs can be carried out independently

Conclusions

- We have reviewed the **state feedback** and **state estimator** design procedure, and showed that the **Separation Principle** holds: the two designs can be carried out independently
- We have discussed the design of **discrete-time** state feedback controllers and observers. The procedures are the same as in continuous-time, save for
  - the choice of desired eigenvalues (which have to be in the unit circle for stability)
  - the compensation of steady state tracking error via the feedforward gain $\mathbf{N}$ (be careful with the formula used)
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