FEL3210 Multivariable Feedback Control

Lecture 4: Performance Limitations in MIMO Systems (Ch.6)

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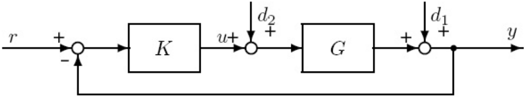
Lecture 3 recap: Introduction to MIMO Feedback



- loop-gain: at output $L = GK$, at input $L_I = KG$
- sensitivity and complimentary sensitivity at output

$$S = (I + L)^{-1}, \quad T = L(I + L)^{-1}$$

  e.g, $e = Sr - Sd_1 - SGd_2 - Tn$

- sensitivity and complimentary sensitivity at input

$$S_I = (I + L_I)^{-1}, \quad T_I = L_I(I + L_I)^{-1}$$

  e.g., $u = -T_I d_2$

Lecture 3 recap: Performance in MIMO Systems

- consider all signals as sinusoids with frequency ω and use 2-norm to quantify amplitude $\|y(\omega)\|_2 = \sqrt{\sum_{i=1}^{l} |y_i|^2}$
- then, with $y = G(s)u$

$$\underline{\sigma}\,(G(i\omega)) \leq \frac{\|y\|_2}{\|u\|_2} \leq \bar{\sigma}\,(G(i\omega))$$

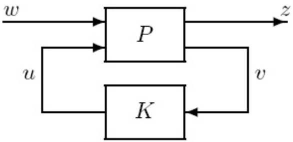
- thus, to bound control error for reference r and disturbance d_1

$$\bar{\sigma}\,(S(i\omega)) \leq \frac{1}{|w_P(i\omega)|} \;\; \forall\omega \quad \Leftrightarrow \quad \|w_P S\|_\infty \leq 1$$

- similarly, for noise

$$\bar{\sigma}(T(i\omega)) \leq \frac{1}{|w_T(i\omega)|} \;\; \forall\omega \quad \Leftrightarrow \quad \|w_T T\|_\infty \leq 1$$

Remark on sinusoids and H_∞-norm



- we assume sinusoid signals throughout, and worst case signals are then, for a generalized plant $z = F(P, K)w$

$$\max_{\omega} \max_{w(\omega)} \frac{\|z(\omega)\|_2}{\|w(\omega)\|_2} = \max_{\omega} \bar{\sigma}\left(F(j\omega)\right) = \|F\|_\infty$$

- but, the H_∞-norm equals the induced 2-norm for any time domain signal

$$\|F\|_\infty = \max_{w(t)\neq 0} \frac{\|z(t)\|_2}{\|w(t)\|_2}$$

Hence, “worst case signal” is always a sinusoid

Outline

Performance limitations in MIMO feedback

- $S + T = I$
- RHP zeros and poles
  - interpolation constraints
  - disturbances and RHP zeros
- The Sensitivity Integral
- Limitations from input constraints
- ”Limitations” from uncertainty

Algebraic Limitation I. $S + T = I$

From *Fan's Theorem*

$$\sigma_i(A) - \bar{\sigma}(B) \leq \sigma_i(A + B) \leq \sigma_i(A) + \bar{\sigma}(B)$$

and $S + T = I$

$$|1 - \bar{\sigma}(T)| \leq \bar{\sigma}(S) \leq 1 + \bar{\sigma}(T)$$

$$|1 - \bar{\sigma}(S)| \leq \bar{\sigma}(T) \leq 1 + \bar{\sigma}(S)$$

Thus, at any frequency ω

- can not make both $\bar{\sigma}(S)$ and $\bar{\sigma}(T)$ small
- $\bar{\sigma}(T) >> 1 \quad \Leftrightarrow \quad \bar{\sigma}(S) >> 1$

Recap: Zero and pole directions

- **Zero direction:** if z is a zero of $G(s)$, then

$$G(z)u_z = 0 \cdot y_z$$

Normalize so that $u_z^H u_z = 1, \quad y_z^H y_z = 1$, then we can also write

$$y_z^H G(z) = 0 \cdot u_z^H$$

- **Pole directions:** if p is a pole of $G(s)$, then

$$G(p)u_p = \infty \cdot y_p$$

If $G^{-1}(p)$ exist, we can also write

$$G^{-1}(p)y_p = 0 \cdot u_p$$

– In the following we assume all zero and pole directions have been normalized to have length 1, e.g., $y_z^H y_z = 1$, $y_p^H y_p = 1$

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Algebraic Limitation II. Interpolation constraints

- If $G(s)$ has a RHP zero at z with output direction y_z, then for internal stability we require

$$y_z^H T(z) = 0 \; ; \quad y_z^H S(z) = y_z^H$$

  - follows from $y_z^H L(z) = 0 \Rightarrow y_z^H T(z) = 0$ $\Rightarrow y_z^H (I - S(z)) = 0$

Thus,

– $T(s)$ must retain any RHP zero and zero direction in $G(s)$

– essentially, $S(z) = 1$ in zero output direction

Algebraic Limitation II. Interpolation constraints

- If $G(s)$ has a RHP pole at p with output direction y_p, then for internal stability we require

$$S(p)y_p = 0\ ; \quad T(p)y_p = y_p$$

  - follows from $L^{-1}(p)y_p = 0$ and $S = TL^{-1}$

Thus,

– $S(s)$ must have RHP zeros where $G(s)$ has RHP poles

– essentially, $T(p) = 1$ in pole output direction

Analytical Constraint I. Minimum peaks from RHP poles and zeros

From the interpolation constraints and Maximum Modulus Thm

- Assume $G(s)$ has a RHP zero at z. Then, with a scalar weight w_P

$$\|w_P S\|_\infty = \max_\omega \bar{\sigma}(w_P S) = \max_\omega |w_p|\bar{\sigma}(S) \geq |w_P(z)|$$

  - generalization of Thm. 5.3 for SISO systems (not considering RHP poles)
  - same restriction on $\bar{\sigma}(S)$ as on $|S|$ in SISO case

- Assume $G(s)$ has a RHP pole at p. Then, with a scalar weight w_T

$$\|w_T T\|_\infty = \max_\omega \bar{\sigma}(w_T T) = \max_\omega |w_T|\bar{\sigma}(T) \geq |w_T(p)|$$

  - generalization of Thm. 5.4 for SISO systems (not considering RHP zeros)
  - require minimum bandwidth for $\bar{\sigma}(T)$ to stabilize system

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Minimum peaks - combined RHP poles and zeros

Theorem 6.1 *consider a rational $G(s)$ with N_z distinct RHP zeros and N_p distinct RHP poles, with corresponding normalized output directions $y_{z,i}$ and $y_{p,i}$, respectively. Then the following tight lower bounds apply*

$$\min_K \|S\|_\infty = \min_K \|T\|_\infty = \sqrt{1 + \bar{\sigma}^2 \left(Q_z^{-1/2} Q_{zp} Q_p^{-1/2} \right)}$$

where

$$[Q_z]_{ij} = \frac{y_{z,i}^H y_{z,j}}{z_i + \bar{z}_j}, \quad [Q_p]_{ij} = \frac{y_{p,i}^H y_{p,j}}{\bar{p}_i + p_j}, \quad [Q_{zp}]_{ij} = \frac{y_{z,i}^H y_{p,j}}{z_i - p_j}$$

- computable **tight bound** for any number of RHP poles and zeros
- minimum peaks depend on distance between z and p as well as the alignment of their directions (no interference if orthogonal directions)

Special case: single RHP pole and zero

For a system $G(s)$ with one RHP pole p and one RHP zero z, Theorem 6.1 yields

$$\min_K \|S\|_\infty = \min_K \|T\|_\infty = \sqrt{sin^2\phi + \frac{|z+p|^2}{|z-p|^2} cos^2\phi}$$

where $\phi = cos^{-1}|y_z^H y_p|$

Example:

$$G(s) = \frac{1}{s+1}\begin{pmatrix} \frac{s+1}{s-1} & s+7 \\ 1 & s+1 \end{pmatrix} \; ; \quad z=2,\; y_z = \begin{pmatrix} -0.32 \\ 0.95 \end{pmatrix},\; p=1,\; y_p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

this yields $\phi = 1.25\; rad$, and

$$\min_K \|S\|_\infty = \min_K \|T\|_\infty = 1.1$$

For SISO plant with $z = 2$ and $p = 1$ we get $\min \|S\|_\infty = \min \|T\|_\infty = 1.73$ $(\phi = 0)$

Moving the constraints from RHP zeros to specific outputs

- the constraint

$$y_z^H T(z) = 0$$

  imposes only l constraints for the l^2 elements $T_{ij}(s)$ of $T(s)$
- thus, there exist some freedom in which elements $T_{ij}(s)$ to restrict in order to satisfy interpolation constraints

Example:

consider example system with RHP zero z and

$$y_z = \begin{pmatrix} -0.32 \\ 0.95 \end{pmatrix}$$

$y_z^H T(z) = 0 \Rightarrow$

$$-0.32\, T_{11}(z) + 0.95\, T_{21}(z) = 0 \quad \wedge \quad -0.32\, T_{12}(z) + 0.95\, T_{22}(z) = 0$$

- with decoupling control

$$T_{12}(s) = T_{21}(s) = 0 \quad \Rightarrow \quad T_{11}(z) = T_{22}(z) = 0$$

  i.e., RHP zero appears in both outputs
- with perfect control of output y_1,

$$T_{11}(s) = 1 \wedge T_{12}(s) = 0 \quad \Rightarrow T_{22}(z) = 0$$

  i.e., RHP zero appears in output y_2 only

Pinned zeros

- the effect of a RHP zero z can be moved to outputs with non-zero elements in the output zero direction, i.e., $y_{z,i} \neq 0$
- a RHP zero z with some elements $y_{z,i} = 0$ is called a **pinned zero**, i.e., it is pinned to outputs with $y_{z,i} \neq 0$

Analytical Constraint II. Sensitivity integral

Assume the loop-gain $L(s)$ has entries with pole excess at least 2, and N_P RHP poles at p_i. Then, for closed-loop stability the sensitivity function must satisfy

$$\int_0^{\infty} \ln|\det S(j\omega)| d\omega = \pi \sum_{i=1}^{N_P} Re(p_i)$$

– essentially, $\det S(s)$ is a sensitivity function with $\det S(\infty) = 1$. The rest then follows from *Cauchy integral theorem* (see Lec.2)

Analytical Constraint II. Sensitivity integral

- for any square matrix

$$|det(S)| = \prod_i \sigma_i(S)$$

hence, the sensitivity integral can be written

$$\sum_i \int_0^\infty \ln \sigma_i \left(S(j\omega)\right) d\omega = \pi \sum_{i=1}^{N_P} Re(p_i)$$

  - interpretation: must make trade-off between frequencies as well as between system directions.

Controllability Analysis

- Given a system $G(s)$ and a set of performance specifications, we would like to analyze if the specifications are feasible.
- The analysis should be independent of the controller $K(s)$, i.e., provide an answer as to whether there exist *any* controller that can meet the specifications.
- The algebraic and analytical constraints presented above are fundamental and must be satisifed by *any* controller.
- Next:
  - functional controllability
  - requirements imposed by disturbances
  - limitations from input constraints
  - limitations from uncertainty

Functional Controllability

Definition: *A system $G(s)$ with m inputs and l outputs is functionally controllable if the normal rank r of $G(s)$ equals the number of outputs l*

- a system with fewer inputs than outputs, $m < l$, has rank $r \leq m < l$ and is hence *functionally uncontrollable*.
- a square $n \times n$ system $G(s)$ is functionally uncontrollable iff $\det G(s) \equiv 0$, i.e., $G(s)$ is singular for all s.

Performance requirements from disturbances

Recall

$$y = Gu + G_d d \quad \Rightarrow \quad e = SG_d d = Sg_{d_1} d_1 + Sg_{d_2} d_2 + \ldots$$

where d_i are scalar disturbances

- performance requirement $\|e(\omega)\|_2 < 1$ implies, for each disturbance d_i

$$\bar{\sigma}(Sg_{d_i}) \leq 1 \ \forall \omega \quad \Leftrightarrow \quad \|Sg_{d_i}\|_\infty \leq 1$$

- define *disturbance direction*

$$y_{d_i} = \frac{g_{d_i}}{\|g_{d_i}\|_2}$$

- requirement becomes

$$\bar{\sigma}(Sy_{d_i}) \leq \frac{1}{\|g_{d_i}\|_2} \ \forall \omega$$

thus, requirement on S is only in the disturbance direction y_{d_i}

Disturbances and directions of S

Consider SVD of *given* sensitivity function, $S = U\Sigma V^H$

$$S\bar{v} = \bar{\sigma}(S)\bar{u}\ , \quad S\underline{v} = \underline{\sigma}(S)\underline{u}$$

- Case 1: disturbance alligned with high-gain direction of S

$$y_{d_i} = \bar{v} \quad \Rightarrow \quad \bar{\sigma}(S) \leq \frac{1}{\|g_{d_i}\|_2} \forall\omega$$

  i.e., requirement is on $\bar{\sigma}(S)$
- Case 2: disturbance alligned with low-gain direction of S

$$y_{d_i} = \underline{v} \quad \Rightarrow \quad \underline{\sigma}(S) \leq \frac{1}{\|g_{d_i}\|_2} \forall\omega$$

  i.e., requirement is on $\underline{\sigma}(S)$

Disturbances and RHP zeros

If $G(s)$ has a RHP zero z, then $y_z^H S(z) = y_z^H$ and

$$\|Sg_{d_i}\|_\infty \geq \|y_z^H S g_{d_i}\|_\infty \geq |y_z^H g_{d_i}(z)|$$

- hence, must require
$$|y_z^H g_{d_i}(z)| < 1$$
recall, for SISO $|g_{d_i}(z)| < 1$
- requirements depend on allignment of y_z and y_{d_i}:
  - if $y_z \perp y_{d_i}$ then $y_z^H g_{d_i} = 0$, i.e., no interference between disturbance and RHP zero
  - if $y_z \parallel y_d$ then $y_z^H g_{d_i} = \|g_d(z)\|_2$ and we require $\|g_{d_i}(z)\|_2 < 1$, as in SISO case

Example: RHP zero and disturbance attenuation

$$G(s) = \frac{1}{0.1s+1}\begin{pmatrix} \frac{s+1}{s-1} & s+7 \\ 1 & s+1 \end{pmatrix} \;; \quad G_d = \frac{1}{0.1s+1}\begin{pmatrix} -0.6 & 50 \\ 1.8 & 16 \end{pmatrix}$$

Zero at $s = 2$ with $y_z^H = \begin{pmatrix} -0.31 & 0.95 \end{pmatrix}$

- For disturbance d_1
$$|y_z^H g_{d_1}(2)| = 1.58$$
- For disturbance d_2
$$|y_z^H g_{d_2}(d)| = 0.25$$

Thus, attenuation of disturbance d_1 not feasible

Limitations imposed by input constraints

Perfect disturbance attentuation

$$y = Gu + g_{d_i} d_i \quad \overset{y=0}{\Rightarrow} \quad u = -G^{-1} g_{d_i} d_i$$

- with $|d_i| \leq 1 \; \forall\omega$ the condition $\|u(\omega)\|_2 < 1 \; \forall\omega$ implies

$$\bar{\sigma}(G^{-1} g_{d_i}) < 1 \; \forall\omega \quad \Rightarrow \quad \|G^{-1} g_{di}\|_\infty < 1$$

- similar for reference tracking, with $G_d = R$

Disturbances and setpoint changes closely alligned with weak output direction $\underline{u}$ of G most difficult

Example:

$$G = \begin{pmatrix} 10 & -11 \\ 11 & -10 \end{pmatrix} \; ; \quad G_d = \begin{pmatrix} 3 & 2 \\ 3 & -2 \end{pmatrix}$$

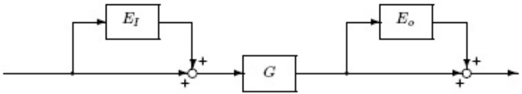
inputs for perfect disturbance attenuation

$$u = G^{-1} G_d = \begin{pmatrix} 0.14 & -2 \\ -0.14 & -2 \end{pmatrix}$$

- disturbance d_2 requires largest inputs despite $\|g_{d_2}\|_2 < \|g_{d_1}\|_2$

"Limitations" imposed by uncertainty

Inputs and outputs are always uncertain

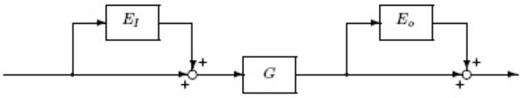


"True" plant

$$G_p = (I + E_o)G(I + E_I)$$

- The uncertainty blocks E_I and E_O can represent physical uncertainty in actuators and sensors, as well as "lumped" model uncertainty
- The blocks E_I and E_O will often have structure, e.g., be diagonal in the case of independent input/output uncertainty
- We typically describe uncertainty in terms of structure and norm bounds on E_I and E_O (more on this later)

Feedforward Control and Uncertainty



$$y = T_r r$$

Consider perfect feedforward control $T_r = I \;\Rightarrow\; K_f = G^{-1}$

- **With output uncertainty** $G_p = (I + E_o)G$ we get

$$T_{rp} = (I + E_o)T_r$$

  i.e., same relative uncertainty as in G

- **With input uncertainty** $G_p = G(I + E_I)$ we get

$$T_{rp} = G(I + E_I)G^{-1} = (I + GE_IG^{-1})T_r$$

  i.e., relative uncertainty becomes GE_IG^{-1}

Feedforward Control and Uncertainty

With input uncertainty

$$T_{rp} = (I + GE_I G^{-1}) T_r$$

- Consider norm of $GE_I G^{-1}$ (at each frequency ω)

$$\|GE_I G^{-1}\|_{i2} \leq \|G(j\omega)\|_{i2} \|E_I(j\omega)\|_{i2} \|G^{-1}(j\omega)\|_{i2}$$

- With $\|\cdot\|_{i2} = \bar{\sigma}(\cdot)$ and $\bar{\sigma}(G^{-1}) = 1/\underline{\sigma}(G)$

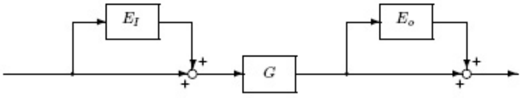
$$\|GE_I G^{-1}\| \leq \|E_I\| \frac{\bar{\sigma}(G)}{\underline{\sigma}(G)} = \|E_I\| \gamma(G)$$

  bound is tight for full block uncertainty E_I
- Thus, for plants with large condition numbers uncertainty at the input "blows up" with feedforward control

Feedforward Control and Uncertainty

- If we restrict the uncertainty block E_I to be diagonal, we can write
$$E_I = D_I E_I D_I^{-1}$$
for any diagonal D_I
- The uncertainty GE_IG^{-1} can then be written
$$(GD_I)E_I(GD_I)^{-1}$$
- This yields
$$\|GE_IG^{-1}\|_{i2} \leq \|E_I\|_{i2} \min_{D_I} \gamma(GD_I) = \|E_I\|_{i2}\gamma^*(G)$$
where γ^* is the minimized condition number
- With $|E_{I,jj}| = \epsilon_j$ the diagonal elements of GE_IG^{-1} are given by
$$[GE_IG^{-1}]_{ii} = \Sigma_{j=1}^{n} \lambda_{ij}(G)\epsilon_j$$
where λ_{ij} are elements of the RGA $\Lambda = G \times (G^{-1})^T$ (see book)

The Loop Gain $L = GK$ and uncertainty



- The effects of uncertainty on feedforward control will also be seen in the loop-gain $L = GK$ of feedback control systems (when we apply decoupling)
- Thus, the effects of output uncertainty should be similar to the SISO case
- The effect of input uncertainty, however, can be severe for systems that are ill-conditioned, i.e., lead to a blow-up of the loop-gain $\bar{\sigma}(L)$

Feedback Control and Output Uncertainty

- With $G_p = (I + E_O)G$

$$
\begin{aligned}
I + G_p K &= I + (I + E_O)GK = \left(I + E_O GK(I + GK)^{-1}\right)(I + GK) \\
&= (I + E_O T)(I + GK)
\end{aligned}
$$

- The sensitivity function

$$S_p = (I + G_p K)^{-1} = S(I + E_O T)^{-1}$$

- The complementary sensitivity $T_p = I - S_p$

$$T_p = (I + S_p E_O)T$$

Thus, like in SISO case, feedback reduces effect of uncertainty when S is "small" (recall Bode's definition of sensitivity $S = (dT/T)/(dG/G)$)

Feedback and Input Uncertainty

With $G_p + G(I + E_I)$; $E_I = diag(\epsilon_i)$, $|\epsilon_i| < |w_I|$

- Apply decoupling control $u = k(s)G^{-1}(s)$, to obtain

$$T(s) = t(s)I \; ; \quad S(s) = (1 - t(s))I \; ; \quad t(s) = \frac{k(s)}{1 + k(s)}$$

- The loop gain becomes

$$G_p K = GK(I + GE_I G^{-1})$$

- The diagonal relative errors of the loop-gain are given by (see above)

$$[GE_I G^{-1}]_{ii} = \sum_{j=1}^{n} \lambda_{ij}(G)\epsilon_j$$

- for closed-loop we get (see book for derivation)

$$\bar{\sigma}(S_p) \geq \bar{\sigma}(S)\left(1 + \frac{|w_I t|}{1 + |w_I t|}\|\Lambda(G)\|_{i\infty}\right)$$

Summary on Effects of Uncertainty

- The loop-gain for MIMO plants highly sensitive to input uncertainty when $\|\Lambda\|_{i\infty} >> 1$ and we try to compensate for strong directionality in controller K
- *Dilemma:* plants that mostly need compensation for strong directionality are also least robust to such compensation!
- Need to make trade-off between nominal performance and robustness.

Heat-exchanger revisited

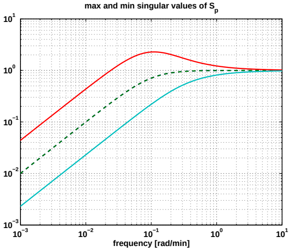
Recall heat-exchanger from lecture 3

$$\begin{pmatrix} T_c \\ T_H \end{pmatrix} = \frac{1}{100s+1} \begin{pmatrix} -18.74 & 17.85 \\ -17.85 & 18.74 \end{pmatrix} \begin{pmatrix} q_C \\ q_H \end{pmatrix}$$

– Relative Gain Array (RGA)

$$\Lambda(G(i\omega)) = \begin{pmatrix} 10.8 & -9.8 \\ -9.8 & 10.8 \end{pmatrix} \quad \Rightarrow \quad \|\Lambda\|_{i\infty} = 20.6$$

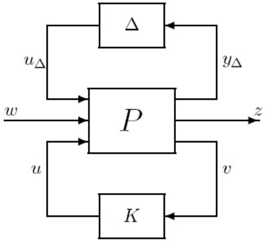
explains severe sensitivity to input uncertainty with decoupler



A procedure for MIMO controllability analysis

1. scale system
2. check for functional controllability, i.e., $r \geq l$?
3. determine poles and zeros in the RHP
4. check minimum peaks for all relevant closed-loop transfer-functions, and determine whether they indicate expected difficulties due to severe peaks
5. compute the RGA to check for (scaling independent) directionality; large RGA elements imply that input uncertainty will restrict achieveable robust performance.
6. determine performance requirements from disturbances and setpoints, e.g., $\|Sy_{d_i}\|_2 < \frac{1}{\|g_{d_i}\|_2} \; \forall\omega$
7. check if RHP zeros and poles prevent acceptable disturbance attenuation
8. check if input constraints prevent acceptable disturbance attenuation

Lecture 5-6: Robust Stability and Robust Performance



- Modeling uncertainty using model sets, e.g.,

$$G_p = \{G + \Delta \mid \|\Delta\|_\infty < w_I\}$$

- Analysis of robust stability and robust performance
- Note! Next lecture is on Tue Mar 6