Sample tasks for the exam / final colloquium in operations research

1. Given the set $C := \{x \in \mathbb{R}^n : Ax \leq b\}$, where A is an $m \times n$ matrix and $b \in \mathbb{R}^m$.

   (a) yes☐ no☐ All elements of matrix A are positive, then $C \neq \emptyset$

   (b) yes☐ no☐ The set $C \neq \emptyset$, then it is a polyhedral set

   (c) yes☐ no☐ The set $C \neq \emptyset$, then it is a polyhedron

   (d) yes☐ no☐ The set $C \neq \emptyset$, then it has at least one vertex

   (e) yes☐ no☐ The set $C \neq \emptyset$ and it has at least one vertex, then C is a convex hull of its vertices

   (f) yes☐ no☐ The set C is convex

   (g) Justify your answer in point (f).

2. Given the following problem:

   A sawmill employing 15 workers in one 8-hour shift produces floorboards and wainscoting boards with 90 m^3 of wood and 1,200 kWh of electricity at its disposal during this shift. The specific consumption of electricity and the required specific number of operating hours are given in the table below.

↓ Consumption per 1 m^3 of wood ⟶	for floorboards	for wainscoting boards
electricity (in kWh)	8	15
man hours (in h)	1,5	1

   The profit from sawing 1 m^3 of wood for floorboards is PLN 120, and for wainscoting boards – PLN 100. The task of the sawmill is to set the production profile in such a way as to achieve maximum profit.

   (a) $\boxed{4}$ Represent this problem as a classical linear programming problem

   m☐mize $z =$

   subject to

   where $\begin{cases} x_1 \text{ denotes} \\ x_2 \text{ denotes} \end{cases}$

   (1)

   (b) $\boxed{2}$ Give the form of the dual problem to problem (P).

   m☐mize $z =$

   subject to

   (2)

(c) $\boxed{3}$ Draw the set of possible solutions for the problem (P).

(d) $\boxed{1}$ Solve problem (P) graphically, indicate the optimal solution on the graph

(e) $\boxed{2}$ How many mł should be wiped onto the floorboards $\boxed{}$, and how much for wainscoting boards $\boxed{}$ to achieve maximum profit?

(f) $\boxed{2}$ What is the maximum profit from sawmill production in one shift? PLN $\boxed{}$

(g) $\boxed{1}$ Provide a brief form of the starting simplex tableau for this problem

(T1)

(h) $\boxed{\pm 1}$ Which phase of the simplex algorithm does this table correspond to? first ☐ second ☐

(i) $\boxed{1}$ In tableau (T1), indicate the main element according to the appropriate phase of the simplex algorithm and transform the tableau.

(T2)

(j) $\boxed{\pm 1}$ yes☐ no☐ Table (T2) shows the feasible solution to problem (P).

(k) $\boxed{1}$ Give the basic solution of problem $(x, u) = \boxed{(\quad,\quad,\quad,\quad)}$, which is presented in tableau (T2).

(l) $\boxed{1}$ Provide the basic solution to problem (D) $(y, v) =$ $\boxed{(\quad , \quad , \quad , \quad)}$, which is presented in tableau (T2).

(m) $\boxed{\pm 1}$ yes☐ no☐ Tableau (T2) shows the optimal solution.

(n) $\boxed{\pm 1}$ yes☐ no☐ Tableau (T2) shows the dually feasible solution.

3. $\boxed{\max 5}$ Given a consistent integer linear programming problem

$$\begin{array}{rrcl} \text{maksymalizować} & c^\top x & & \\ \text{przy ograniczeniach} & Ax & \leq & b \\ & x & \geq & 0 \\ & x & \in & \mathbf{Z}^n \end{array} \tag{PC}$$

where $c, x \in \mathbb{R}^n$, A is an $m \times n$ matrix with positive elements and $b \in \mathbb{R}^m$, $b, c \geq 0$.

(a) $\boxed{\pm 1}$ yes☐ no☐ Problem (PC) always has a feasible solution

(b) $\boxed{\pm 1}$ yes☐ no☐ Problem (PC) always has an optimal solution

(c) $\boxed{\pm 1}$ yes☐ no☐ Problem (PC) can be solved using the simplex method

(d) $\boxed{\pm 1}$ yes☐ no☐ Problem (PC) can be solved by Gomory's cuts

(e) $\boxed{\pm 1}$ yes☐ no☐ If the integer assumption is removed in the problem (PC), the optimal value of the objective function will decrease.

4. Given is a complete graph $G = (V, E)$, the vertices of which are located on the $\mathbb{R}^2$ plane and have coordinates $(1, 0)$, $(0, 1)$, $(0, 2)$, $(3, 4)$, $(3, 6)$, $(5, 1)$ and $(6, 3)$. A weight function $c : E \to \mathbb{R}$ is defined on the set of edges E in the following way: $c(e)$ s equal to the Euclidean distance between the vertices connected by edge e, $e \in E$. Use the Kruskal method to determine the shortest spanning tree of this graph, marking the edges determined by this method one by one.

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