Operations research tasks – list 1

1. (*Diet issue*) A child needs at least 120 units of vitamin A, 60 units of vitamin D, 36 units of vitamin C and 180 units of vitamin E per week. These vitamins are contained in two products P_1 and P_2. Since excess vitamin A may be harmful to the child, it should be provided it in an amount of at most 240 units. Vitamin contents per product unit and product prices are given in the table below.

	vitamin contents per kg	
vitamins	P_1	P_2
A	6	3
D	1	3
C	9	1
E	6	6
price per kg	12	18

You should purchase P_1 and P_2 products in quantities that ensure your weekly requirement for vitamins, for which the purchase cost is the lowest. Present the above problem as a linear programming problem. Present geometrically the set of feasible solutions and objective function contours and determine geometrically the optimal solution.

2. Aluminum and copper are used in the production of P_1 and P_2 goods (counted in pieces). Milling machines and lathes are used to process elements (see table):

resources	quantity	consumption per 1 piece of product	
		P_1	P_2
aluminium (kg)	570	10	70
cooper (kg)	420	20	50
lathes (man hours)	5600	300	400
milling machines (man hours)	3400	200	100
profit (PLN/pcs)		300	800

Determine the amount of production of goods P_1 and P_2 that maximizes profit. Present the above problem as a linear programming problem. Present geometrically the set of feasible solutions and objective function contours and determine geometrically the optimal solution.

3. For its production, the steelworks requires coal with a phosphorus content of no more than 0.03% and a sulfur content of no more than 0.3%. There are three types of coal available on the market at prices of PLN 90, 90, 135 (PLN per tonne) with phosphorus content of 0.06%, 0.04% and 0.02%, and sulfur - 0.2%, 0.4% and 0.3%, respectively. Three types of coal should be mixed to obtain the lowest price and not to exceed the permissible phosphorus and sulfur contents. Present the above problem as a linear programming problem.

4. From paper bales (semi-finished products) 210 cm wide, at least 30 rolls 62 cm wide, 60 rolls 55 cm wide, 60 rolls 40 cm wide should be produced. The manufacturer's task is to minimize the number of bales used so that the specified numbers of individual rolls produced are maintained. Formulate the above problem as an integer linear programming problem. Tip: Designate all types of cutting the bale into rolls of a given width, for which the rest is waste (so-called effective cutting). What can be taken as decision variables? How to define the objective function and constraints?

5. Two sugar refineries are conducting a sugar campaign with the goal of processing a total of at least $3,200$ tons of beets. The daily throughput of the first sugar factory is 100 tons and the second one is 160 tons of beets. It is known that during a sugar campaign, losses occur depending on the duration of the campaign, which can be described by the function $f(t_1, t_2) = 10t_1 + 15t_2$, where t_1 is the duration (in days) of the campaign in the first sugar factory, and t_2 – in the second sugar factory. sugar factory. The first sugar factory can accept a maximum of $1,600$ tons of beets, while the second one can operate for a maximum of 20 days.

   (a) a. How long should the sugar campaign last in each sugar factory to minimize losses?
   (b) b. How to optimally distribute these $3,200$ tons of beets among sugar refineries?
   (c) Formulate the above tasks as mathematical programming tasks.

6. The sawmill received an order to make at least 300 sets of beams. Each set consists of 7 beams 0.7 m long and 4 beams 2.5 m long. The order should be placed so that the waste resulting from cutting 5.2 m long logs is minimal. Formulate the above problem as an integer linear programming problem.

7. In a plant with multi-shift work organization, there is the following minimum demand for employees: 3 people from 00:00 to 4:00, 8 people from 04:00 to 08:00, 10 people from 08:00 to 12:00, 8 people from 12:00 to 16:00, 14 people from 16:00 to 20:00, 5 people from 20:00-24:00. Work for a single employee starts at 0:00, 4:00, 8:00, 12:00,

16:00, 20:00 and lasts 8 hours. Formulate an integer linear programming problem consisting in minimizing the total number of employees in the plant under given constraints. Tip: Consider what to take as decision variables in this problem? How to define the objective function and constraints?

8. An airline company has n types of aircraft that serve m routes. The following quantities are known:

   (a) p_j – flight price for one person on the jth route,

   (b) k_{ij} – flight costs of type i aircraft on the jth route,

   (c) a_i – number of type i aircraft owned,

   (d) c_{ij} – permissible number of passengers in an aircraft of type i on the jth route,

   (e) q_j – expected number of passengers on the j-th route.

   Formulate an integer linear programming problem involving the creation of a flight plan in which, given the given constraints, the airline company's profit is maximum. Tip: As decision variables, take the number of aircraft of particular types intended to serve particular routes. In order to formulate this task as a ZPL, however, certain auxiliary variables should be introduced to express the number of passengers actually transported on individual routes.

9. (Optimal basket of goods) The consumer has cash in the amount of PLN 100 and can use it to purchase two goods P_1 (bread) and P_2 (milk), for which the prices are PLN 1.50/kg and PLN 2.0/l, respectively. The consumer is assigned a utility function $u : \mathbb{R}^2_+ \to \mathbb{R}$, where $u(x, y) = xy$ means his degree of satisfaction resulting from the purchase of x kg of bread and y l of milk. The consumer's goal is to purchase (within his financial capabilities) the quantities of bread and milk for which the utility is greatest. Present the described task as a mathematical programming problem and solve it graphically. How will the solution change when the utility function takes the form $u(x, y) = \min\{x, y\}$.

10. (*Optimal stock portfolio*) The investor intends to create a portfolio consisting of two shares A_1 and A_2, having average rates of return $R_1 = 0.1$ and $R_2 = 0.2$, respectively, and risk $s_1 = 0.1$ and $s_2 = 0.3$. The correlation between these stocks is $\rho_{12} = -0.5$. The investor's utility function has the form $u(s, R) = -s + 2R$. The investor should create a portfolio that maximizes the utility function. Formulate the above problem as a constrained minimization problem. Make an appropriate drawing showing the set of possible solutions and the objective function contours and determine the optimal solution geometrically.