## Operations research tasks - list 4

1. Solve the following tasks using the simplex method. In example (a), select the first (second) column in the starting table as the main column. What points in the set of possible solutions do subsequent simplex tables correspond to? What form do the sets of optimal solutions take? In examples (a) - (c) choose the pivot also for the form of a long simplex array.
(a)

$$
\begin{array}{rr}
\operatorname{maximize} & x_{1}+2 x_{2} \\
\text { subject to } & x_{1}+x_{2} \leq 100 \\
6 x_{1}+9 x_{2} \leq 720 \\
& x_{2} \leq 60 \\
x_{1}, x_{2} \geq 0
\end{array}
$$

(b)
maximize $\quad 4 x_{1}+30 x_{2}$ ubject to $\begin{aligned} 2 x_{1}+x_{2} & \leq 12 \\ 2 x_{1}+3 x_{2} & \leq 18\end{aligned}$ $x_{1}, x_{2} \geq 0$

| maximize | $x_{1}+x_{2}$ |
| ---: | ---: |
| subject to | $x_{1}+x_{2} \leq 10$ |
| $2 x_{1}+x_{2} \leq 16$ |  |
| $x_{1} \leq 6$ |  |
|  | $x_{1}, x_{2} \geq 0$ |

$$
\begin{array}{rr}
\operatorname{maximize} & 2 x_{1}+4 x_{2}+3 x_{3} \\
\text { subject to } & x_{1}+4 x_{2}+3 x_{3} \leq 240 \\
& 2 x_{1}+x_{2}+5 x_{3} \leq 300  \tag{d}\\
& x_{1}+x_{2}+x_{3} \leq 200 \\
& x_{1}, x_{2}, x_{3} \geq 0
\end{array}
$$

2. Using the simplex method, determine all solutions of the following linear programming problems:

$$
\begin{array}{rr}
\operatorname{maximize} & -3 x_{1}+3 x_{2}+x_{3} \\
\text { subject to } & -9 x_{1}+4 x_{2}-18 x_{3} \leq 4 \\
& -10 x_{1}+6 x_{2}+6 x_{3} \leq 6  \tag{b}\\
& -4 x_{1}-3 x_{2}+x_{3} \leq 12 \\
& x_{1}, x_{2}, x_{3} \geq 0
\end{array}
$$

$$
\operatorname{maximize} \quad-x_{1}+x_{2}
$$

$$
\text { subject to }-2 x_{1}+x_{2} \leq 2
$$

$$
-x_{1}+x_{2} \leq 4
$$

$$
x_{1}-3 x_{2} \leq 3
$$

$$
x_{1}, x_{2} \geq 0
$$

(c)

$$
\begin{array}{rr}
\operatorname{maximize} & 54 x_{1}+10 x_{2}-15 x_{3} \\
\text { subject to } & 10 x_{1}+5 x_{2}+4 x_{3} \leq 10 \\
162 x_{1}+30 x_{2}-45 x_{3} \leq 14  \tag{d}\\
& -2 x_{1}-6 x_{2}-5_{3} \leq 1 \\
x_{1}, x_{2}, x_{3} \geq 0
\end{array}
$$

$$
\begin{array}{rr}
\operatorname{maximize} & 6 x_{1}+4 x_{2}-4 x_{3} \\
\text { subject to } & x_{1}+2 x_{2}+2 x_{3} \leq 14 \\
3 x_{1}+2 x_{2}-2 x_{3} \leq 6 \\
& 15 x_{1}+6 x_{2}+10 x_{3} \leq 90 \\
& 75 x_{1}+54 x_{2}-10 x_{3} \leq 282 \\
& x_{1}, x_{2}, x_{3} \geq 0
\end{array}
$$

$$
\begin{array}{rr}
\operatorname{maximize} & 8 y_{1}+10 y_{2}+8 y_{3}+14 y_{4}+5 y_{5}+3 y_{6} \\
\text { subject to } & y_{1}+y_{6} \leq 1 \\
y_{1}+y_{2} \leq 1 \\
y_{2}+y_{3} \leq 1 \\
y_{3}+y_{4} \leq 1 \\
y_{4}+y_{5} \leq 1 \\
y_{5}+y_{6} \leq 1 \\
& y_{1}, y_{2}, y_{3}, y_{4}, y_{5}, y_{6} \geq 0
\end{array}
$$

3. Solve the following linear programming problems using the two-phase simplex method and using the artificial variables method.

$$
\begin{array}{rrrr}
\text { maximize } & x_{1}+2 x_{2} & \text { maximize } & x_{1}+x_{2}+3 x_{3}  \tag{b}\\
\text { subject to } & -3 x_{1}+4 x_{2} \leq 2 & \text { subject to } & x_{1}-x_{2} \geq-10 \\
& 2 x_{1}+x_{2} \geq 4 & \text { (b) } & x_{1}-x_{3} \geq 12 \\
& 2 x_{1}-x_{2} \leq 2 & & -x_{1}+x_{2}+x_{3} \geq-8 \\
& x_{1}, x_{2} \geq 0 & & 2 x_{1}-x_{2}+x_{3} \geq 2 \\
& & & x_{1}, x_{2}, x_{3} \geq 0
\end{array}
$$

4. Solve the dual problem to the problem in task 2(e) using the two-phase simplex method and using the dual simplex method. Pay attention to the connections of this task with task 7 from list 1.
5. Solve the following linear programming problem using the simplex method and present this solution graphically.

$$
\begin{array}{rrl}
\operatorname{maximize} & 100 x_{1}+10 x_{2}+x_{3} & \\
\text { subject to } & x_{1} & \leq 1 \\
& 20 x_{1}+x_{2} & \leq 100 \\
& 200 x_{1}+20 x_{2}+x_{3} & \leq 10000 \\
x_{1}, x_{2}, x_{3} & \geq 0
\end{array}
$$

