Operations research tasks – list 5

1. The following simplex tableau are given:

A

1	$-x_1$	$-x_2$	$-x_3$	
5	-1	-2	-3	z
2	1	3	-2	u_1
0	2	5	-1	u_2
1	3	4	0	u_3

B

1	$-u_1$	$-x_2$	$-x_3$	
3	2	-5	0	z
8	-2	2	4	x_1
3	3	1	1	u_2
15	5	3	5	u_3

C

1	$-x_1$	$-u_2$	$-x_3$	
-5	2	1	4	z
3	1	-1	3	u_1
-2	1	2	0	x_2
1	0	1	-1	u_3

D

1	$-u_3$	$-u_1$	$-x_3$	
8	2	0	3	z
4	-2	-1	4	x_2
3	0	1	2	u_2
0	0	-2	1	x_1

E

1	$-u_1$	$-x_2$	$-u_2$	
8	2	0	1	z
1	1	-2	1	x_1
0	2	2	2	x_3
3	1	-1	3	u_3

F

1	$-x_1$	$-x_2$	$-x_3$	
0	6	12	8	z
-4	-2	-2	0	u_1
1	1	2	2	u_2
-8	-1	3	-4	u_3

(a) In tableau B, indicate the pivot according to the rules of phase II of the simplex algorithm, transform the tableau and present the next simplex tableau:

B^+ :

(b) In table F, indicate the pivot according to the rules of the dual simplex algorithm.

(c) Complete the following table:

Table: $\longrightarrow$	A		B		C		D		E		F	
$\downarrow$	tak	nie	tak	nie	tak	nie	tak	nie	tak	nie	tak	nie
indicates that it is missing a feasible solution	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
presents a basics feasible solution	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
presents a basic solution which is originally degenerate	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
presents a basic solution which is dually degenerate	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
presents a basic optimal solution	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
indicates that there is more optimal solutions	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
indicates that there are no optimal solutions	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
indicates that the set of optimal solutions is unbounded	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
In case the tableau presents an optimal solution, provide:	× × ××		× × ××		× × ××		× × ××		× × ××		× × ××	
the optimal solutions of the primal problem $x^* =$												
the optimal solutions of the dual problem $y^* =$												

2. Solve task 4 from list 1 using the dual simplex method.

3. Let A be a matrix of type $m \times n$, $c \in \mathbb{R}^n$, $b \in \mathbb{R}^m$. Check whether the following linear programming tasks are mutually dual:

   (a)
$$\begin{array}{rr} \text{maximize} & c^\top x \\ \text{subject to} & Ax \leq b \\ & x \geq 0. \end{array} \tag{P1}$$

   i
$$\begin{array}{rr} \text{minimize} & b^\top y \\ \text{subject to} & A^\top y \geq c \\ & y \geq 0. \end{array} \tag{D1}$$

   (b)
$$\begin{array}{rr} \text{maximize} & c^\top x \\ \text{subject to} & Ax = b \\ & x \geq 0. \end{array} \tag{P2}$$

   i
$$\begin{array}{rr} \text{minimize} & b^\top y \\ \text{subject to} & A^\top y \geq c \end{array} \tag{D2}$$

   (c)
$$\begin{array}{rr} \text{maximize} & c_1^\top x_1 + c_2^\top x_2 \\ \text{with respect to} & (x_1, x_2) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \\ \text{subject to} & A_{11}x_1 + A_{12}x_2 = b_1 \\ & A_{21}x_1 + A_{22}x_2 \leq b_2 \\ & x_1 \geq 0 \end{array} \tag{P3}$$

   i
$$\begin{array}{rr} \text{minimize} & b_1^\top y_1 + b_2^\top y_2 \\ \text{with respect to} & (y_1, y_2) \in \mathbb{R}^{m_1} \times \mathbb{R}^{m_2} \\ \text{subject to} & A_{11}^\top y_1 + A_{21}^\top y_2 \geq c_1 \\ & A_{12}^\top y_2 + A_{22}^\top y_2 = c_2 \\ & y_2 \geq 0, \end{array} \tag{D3}$$

   where the block matrix
$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$
   is of type $(m_1 + m_2) \times (n_1 + n_2)$, the vectors $c_1 \in \mathbb{R}^{n_1}, c_2 \in \mathbb{R}^{n_2}, b_1 \in \mathbb{R}^{m_1}, b_2 \in \mathbb{R}^{m_2}$

4. Using the complementarity theorem for the primary problem (**??**) and the dual problem (**??**), show that the following theorem holds:

   Theorem. Let x and y be feasible solutions for the primary problem (**??**) and the dual problem (**??**). Then x and y are optimal solutions if and only if
$$x^\top (c - A^\top y) = 0.$$

5. Given a linear programming problem
$$\begin{array}{rr} \text{minimize} & 3x_1 + x_2 + 9x_3 + x_4 \\ \text{subject to} & x_1 + 2x_3 + x_4 = 4, \\ & x_2 + x_3 - x_4 = 2, \\ & x_1, x_2, x_3, x_4 \geq 0. \end{array}$$

   It is known that $x^* = (0, 6, 0, 4)$ is a solution to this problem. Determine a solution to the dual problem using the complementarity theorem given in the task 4.

6. Using the duality theorems, reduce the linear programming problem to determining a solution of a certain system of linear inequalities.