Operations research tasks – list 6

1. The following linear programming problem is given

$$\begin{array}{rr} \text{maximize} & ax_1 + bx_2 \\ \text{subject to} & x_1 - x_2 \le 3, \\ & -2x_1 + x_2 \le 5, \\ & x_1, x_2 \ge 0, \end{array}$$

where $a, b \in \mathbb{R}$. Formulate the dual problem. Draw the areas of feasible solutions of the primary and dual problems. For what constants a, b:

(a) does this problem have exactly one optimal solution?
(b) does this problem have infinitely many optimal solutions?
(c) does this problem have no optimal solutions?
(d) is the set of optimal solutions boundeded?
(e) any optimal solution is a convex combination of the basic optimal solutions?

Hint: draw the set of possible solutions. Note that it is enough to limit ourselves to vectors (a, b) from the unit sphere in $\mathbb{R}^2$. Determine such vectors in the direction of which the maximum is realized by points, segments, and rays.

2. Given a transportation problem with 3 warehouses with supply 6, 2, 10 and 4 stores with demand 7, 5, 3, 3. The cost matrix has the form

$$C = \begin{bmatrix} 10 & 3 & 11 & 7 \\ 4 & 2 & 6 & 1 \\ 5 & 8 & 15 & 9 \end{bmatrix}$$

(a) Is this problem balanced?
(b) Designate the initial transport plan according to:
   1. the northwest angle method,
   2. the method of the minimum element of the cost matrix,
   3. Vogel's approximation method.
(c) For each of the designated transport plans, determine the corresponding values of the objective function.
(d) For each of the designated transport plans, determine the dual variables, then the negative reduced costs, and finally draw the base loop.
(e) Present on the transport table a new transport plan resulting from the transformations made in point (d).
(f) Does the new table represent an optimal transport plan? Justify your answer.
(g) Schematically draw a transport plan in the form of a graph that corresponds to the table presented in point (e).

3. There is an assignment problem in which 5 employees are to be assigned 5 activities, for which a cost matrix is given

$$C = \begin{bmatrix} 1 & 2 & 5 & 3 & 2 \\ 3 & 1 & 4 & 2 & 1 \\ 6 & 4 & 3 & 2 & 2 \\ 4 & 2 & 2 & 1 & 3 \\ 5 & 3 & 3 & 2 & 1 \end{bmatrix},$$

where c_{ij} is the cost of assigning the ith activity to the jth employee, $i, j = 1, 2, 3, 4, 5$.

(a) Rozwiązać to zadanie przy pomocy algorytmu transportowego.
(b) Przedstawić graficznie przykładowy dopuszczalny plan przydziału.

4. Given is a complete graph $G = (V, E)$, the vertices of which are located on the $\mathbb{R}^2$ plane and have coordinates (1,0), (0,1), (0,2), (3,4), (3,6), (5,1) and (6,3). On the set of edges E, the weight function $c : E \to \mathbb{R}$ is defined as follows: $c(e)$ is equal to the Euclidean distance between the vertices connected by edge e, $e \in E$.

(a) Determine the shortest spanning tree of this graph marking the edges determined by this method one by one:
   1. by the Kruskal method,
   2. by the Prim method.
(b) Use the Ford method to determine the shortest paths from the starting point (1,0) to any point of this graph.